Short explanation of nTMM physics

Newton equations describe the trajectory

of particle motion as $f(\boldsymbol{x}, t)$

Schrödinger equation in QM describes evolution of a particle (or a system) by a wave function $\psi(\boldsymbol{x}, t)$

$$i\hbar\frac{\partial}{\partial t}\psi = \widehat{\mathrm{H}}\psi$$

$\widehat{H} = i\hbar\frac{\partial}{\partial t}$ - operator of energy (= Hamiltonian) including

$E = mc^2 + K.E. + P.E.$ defines state of the system at $(\boldsymbol{x}, t)$

Non-trivial: $\widehat{H}$ might include absorption

Operator means: $\widehat{H}\psi = E\psi$

ψ in QM is a “wave of probability “

$= $ complex function $\psi = a + ib$

Probability of particle state (Real) is described by ψ (complex)

$$P(x, t) = \psi\psi^{*} \equiv |\psi|^{2} = (a + ib)(a - ib) = a^{2} + b^{2}$$

for free neutron when total energy E is conserved

$$i\hbar \frac{d}{dt}\psi = E\psi$$

and solution of S.E. can be found from

$$i\hbar \frac{d\psi}{\psi} = Edt$$

$$\psi = Ae^{-\frac{i}{\hbar}Et} = A\left(cos\frac{Et}{\hbar} - i\ sin\frac{Et}{\hbar}\right)$$ called plane wave solution

If neutron is mixed with mirror neutron, we describe that as

$$\psi = \begin{pmatrix} \psi_n \\ \psi_{n'} \end{pmatrix} \equiv \begin{pmatrix} n \\ n' \end{pmatrix}$$

n and n' are wave functions of $\boldsymbol{x}$ and t

In this case Schrödinger equations :

$$i\hbar\begin{pmatrix}\dot{n}\\ \dot{n}'\end{pmatrix}=\begin{pmatrix}E_n & \epsilon\\ \epsilon & E_{n'}\end{pmatrix}\begin{pmatrix}n\\ n'\end{pmatrix}$$

Unknown coupling constant

for the mixed state of neutron $\hat{H}$ is a matrix : $\hat{H}=\begin{pmatrix}E_n & \epsilon\\ \epsilon & E_{n'}\end{pmatrix}$

These are two coupled Schrödinger equations :

$$\begin{cases} i\hbar\dot{n} = E_n n + \epsilon n' \\ i\hbar\dot{n}' = \epsilon n + E_{n'} n' \end{cases}$$

We cannot find plane wave solution for a mixture of two wave functions n and n'. It would be possible if Hamiltonian will be diagonal.

Following Linear Algebra recipe, one can find linear combination of two wave functions n and n' that will make Hamiltonian diagonal

for $\binom{n}{n'}$ $\hat{H}$ is non-diagonal ; for functions $\binom{n_1}{n_2}$ $\hat{H}$ will be diagonal

States n_1 and n_2 will have certain fixed energies E_1 and E_2 needed for two plane wave solutions

$$\begin{cases} n_1 = +n\, cos\theta + n' sin\theta \\ n_2 = -n\, sin\theta + n' cos\theta \end{cases} \qquad \begin{cases} n \;= +n_1\, cos\theta - n_2 sin\theta \\ n' = +n_1\, sin\theta + n_2 cos\theta \end{cases}$$

At what angle θ it will be possible? $\Rightarrow$ $\tan 2\theta = \dfrac{2\epsilon}{\Delta E}$, $\Delta E = E_1 - E_2$

For this mixing θ one can find two plane wave solutions for n_1 and n_2 and then using the right equations above find the solutions for n and n'.

Modulus square of wave functions n and n' will give probabilities of observation of these states at time t

$$P_{n'}(t) = |n'(t)|^2 = \frac{\epsilon^2}{\omega^2} sin^2(\omega t/\hbar); \qquad \omega^2 = \left(\frac{\Delta E}{2}\right)^2 + \epsilon^2$$

$P_n(t) = 1 - P_{n'}(t)$ that is oscillation between states n and n'

Now, let's make $\Delta E = 0 \quad \rightarrow \quad \omega^2 = \epsilon^2; \quad P_{n'}(t) = sin^2(\epsilon t/\hbar) \cong (\epsilon t/\hbar)^2$

Potentially can grow to 1, i.e. 100%, but in our case small

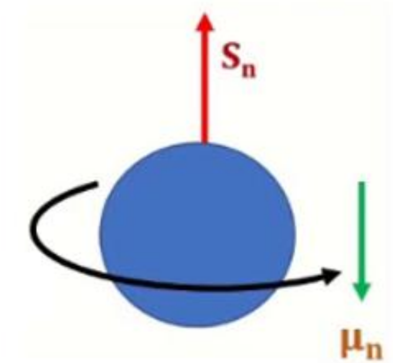
maximum possible oscillation determined by ϵ

Usually, $\Delta E/2 \gg \epsilon$ and then $P_{n'}(t) \approx \left(\frac{2\epsilon}{\Delta E}\right)^2 sin^2(\Delta E t/2\hbar)$

oscillations are present, are frequent, but strongly suppressed by ΔE .

That is how $n \leftrightarrow n'$ oscillation can happen without nTMM.

With nTMM



Due to motion of quarks inside the neutron,

neutron has a “dipole magnetic moment” $\boldsymbol{\mu}$

that in magnetic field $\boldsymbol{B}$ makes

a potential energy $V = \boldsymbol{\mu} \cdot \boldsymbol{B}$

$$\boldsymbol{\mu} = 60.3\ neV/T = 6.03 \times 10^{-12}\ eV/G$$

As QM particle with spin $1/2$,

neutron in magnetic field can have two states of P.E.

$+\mu B$ and $-\mu B$

Neutron’s magnetic moment $\boldsymbol{\mu}$ can be very precisely measured but theoretically, from the first principles, its value cannot be predicted by QCD better than with 0.1%

If neutron for a small part consists of (mixed with)

mirror neutron, $\boldsymbol{\mu}$ can be modified by including

$\boldsymbol{\eta}$ - "nTMM" due to existence of $n \leftrightarrow n'$

We are talking about $\boldsymbol{\eta}$ that can be a small fraction κ of $\boldsymbol{\mu}$:

$$\kappa = \boldsymbol{\eta}/\boldsymbol{\mu} < 10^{-5}$$

Similarly, the mirror neutron for a small part would consists of (mixed with) ordinary neutron, and $\boldsymbol{\mu}'$ can be modified by including $\boldsymbol{\eta}'$ - nTMM due to existence of $n \leftrightarrow n'$: $\boldsymbol{\mu} = \boldsymbol{\mu}'$ and $\boldsymbol{\eta} = \boldsymbol{\eta}'$

That means that through nTMM mirror neutron will be sensitive to our magnetic field !

Hamiltonian without nTMM

$$\hat{H} = \begin{pmatrix} E_n & \epsilon \\ \epsilon & E_{n'} \end{pmatrix}$$

Hamiltonian with nTMM

$$\hat{H} = \begin{pmatrix} E_n & \epsilon + \kappa \boldsymbol{\mu} \cdot \boldsymbol{B} \\ \epsilon + \kappa \boldsymbol{\mu} \cdot \boldsymbol{B} & E_{n'} \end{pmatrix}$$

That means that ϵ determining maximum probability of oscillation can be increased to effective $(\epsilon + \kappa \boldsymbol{\mu} \cdot \boldsymbol{B})$, that is it can be enhanced by our magnetic field $\boldsymbol{B}$

Well, maximum possible oscillation probability can be increase by magnetic field B, but B will also define the P.E. for E_n , $\mathrm{i.e.}$ ΔE will be increased and oscillation will be suppressed.

$$\Delta E = (K.E. \pm \boldsymbol{\mu} \cdot \boldsymbol{B})_n - (K.E.)_{n'} = \pm \boldsymbol{\mu} \cdot \boldsymbol{B}$$ and suppression factor $$\left(\frac{2\epsilon}{\Delta E}\right)^2$$

We need to zero ΔE. For that we make to oscillation process to happen in gas together with positive Fermi ("optical") potential V_F.

Then total potential energy for half of the neutron beam can be $P.E. = V_F - \boldsymbol{\mu} \cdot \boldsymbol{B}$ ($+\boldsymbol{\mu} \cdot \boldsymbol{B}$ will not do the job) and we can zero the ΔE while increasing oscillation probability due to nTMM.

0

$$\omega_{nTMM}^2 = \left(\frac{V_F - \boldsymbol{\mu}\cdot\boldsymbol{B}}{2}\right)^2 + (\epsilon + \kappa\boldsymbol{\mu}\cdot\boldsymbol{B})^2$$

1

$$P_{n'}(t) = \frac{(\epsilon+\kappa\boldsymbol{\mu}\cdot\boldsymbol{B})^2}{\omega^2}\, sin^2(\omega t/\hbar)$$

Formulas for any magnetic field

when we make magnetic field exactly compensating $V_F \implies resonance$

at the resonance value of magnetic field, we'll have:

$$P_{n\to n'}(t) = sin^2[(\epsilon + \kappa\boldsymbol{\mu}\cdot\boldsymbol{B})t/\hbar] \approx \kappa^2(\boldsymbol{\mu}\cdot\boldsymbol{B}\, t/\hbar)^2$$

here t is the time of flight in uniform magnetic field compensating V_F

We chose max possible $B = 26$ G corresponding to CO_2 at 1.3 atm

Since we cannot detect n', we do regeneration experiment, where all not used neutrons after 1-st magnet are absorbed and produced mirror neutrons are transformed back to observable neutrons.

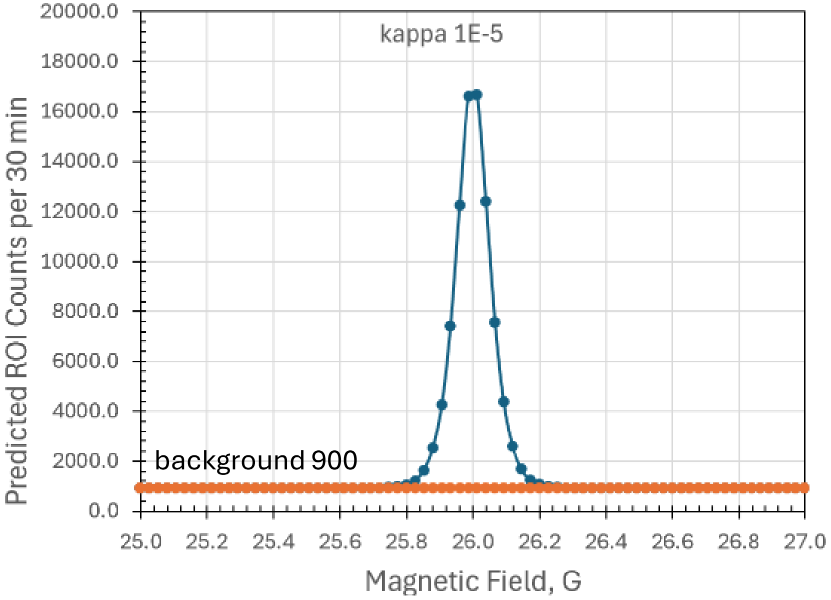
$$P_{n'\to n}(t) \approx \kappa^2 (\boldsymbol{\mu} \cdot \boldsymbol{B}\; t/\hbar)^2$$

and total observation probability:

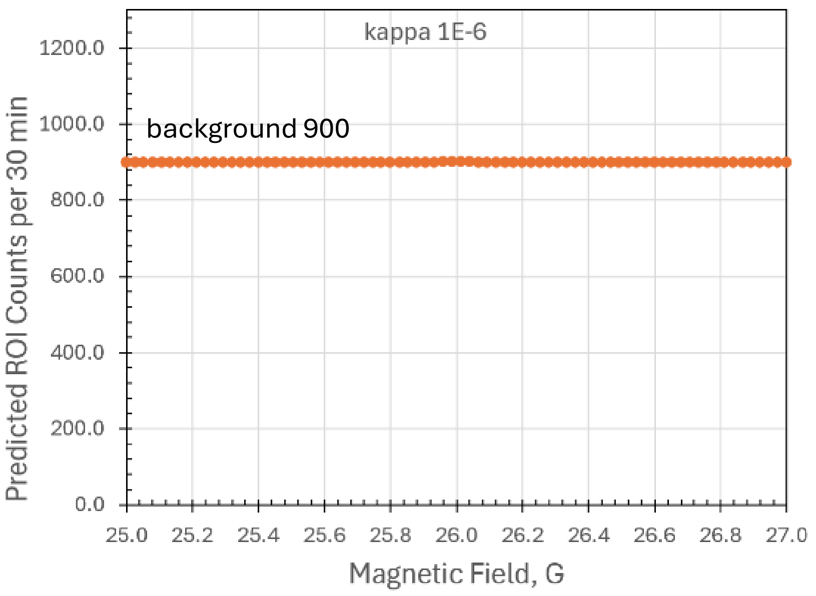
$$P_{n\to n'\to n}(t) \approx \kappa^4 (\boldsymbol{\mu} \cdot \boldsymbol{B}\; t/\hbar)^4$$

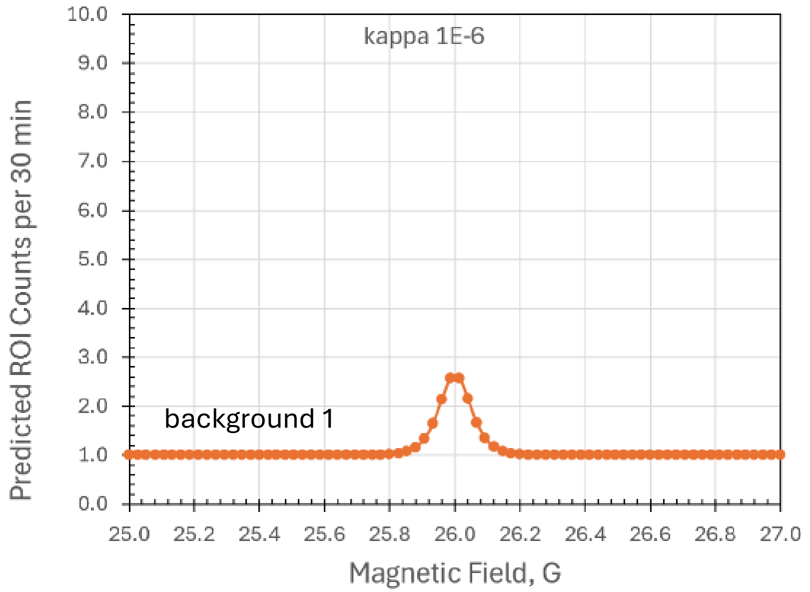
Time of flight through a magnet depends on velocity. Therefore, prediction of probability per neutron should be averaged over the spectrum of neutron velocities in the beam.

kappa 1E-5



kappa 1E-6





Prediction of nTMM effect in GPSANS for 30 min runs

We can set a limit $\kappa(nTMM) \lesssim 3 \times 10^{-6}$

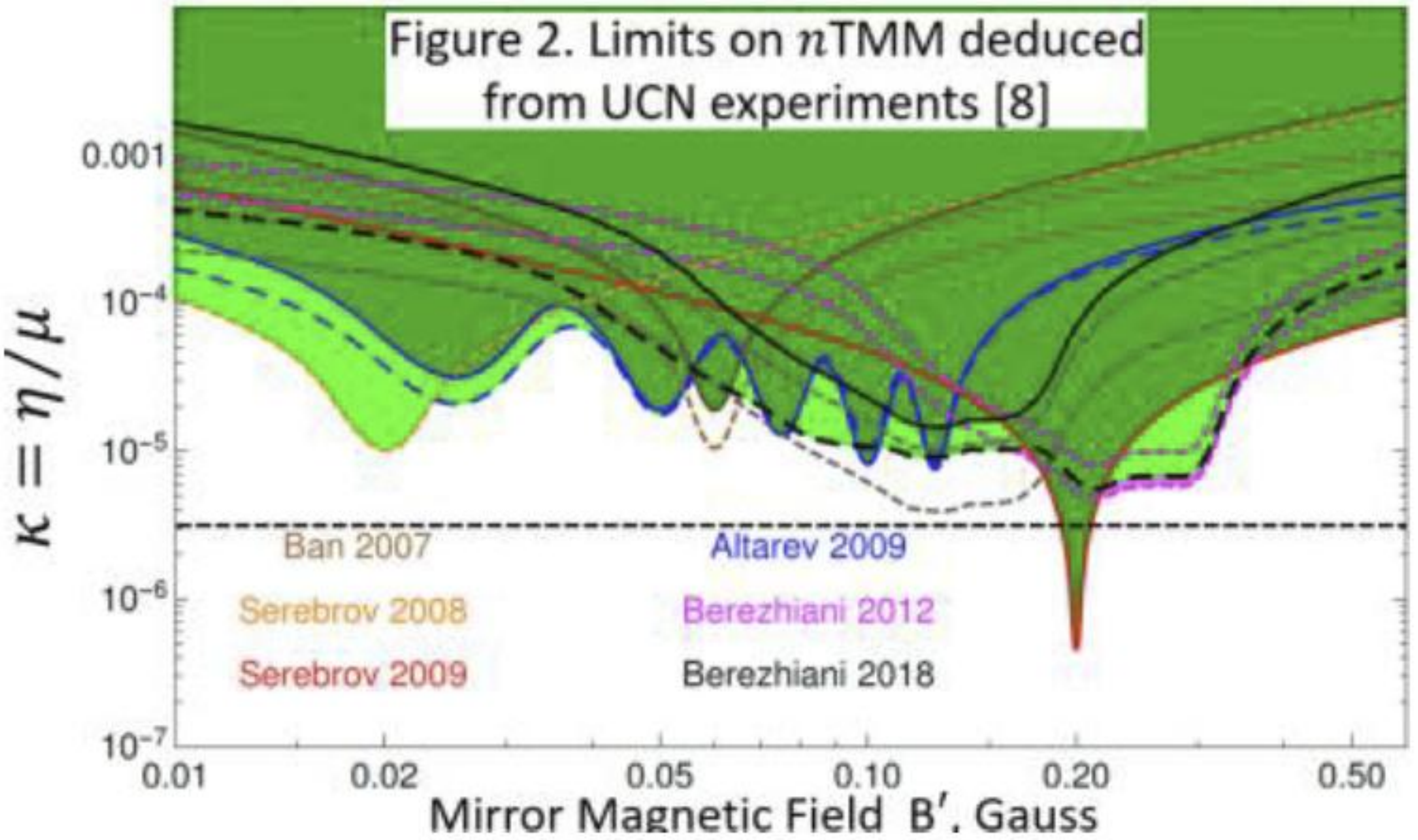


Figure 2. Limits on nTMM deduced from UCN experiments [8]
0.001
10^{-4}
10^{-5}
10^{-6}
10^{-7}
$\kappa \equiv \eta/\mu$
Ban 2007
Serebrov 2008
Serebrov 2009
Altarev 2009
Berezhiani 2012
Berezhiani 2018
0.01
0.02
0.05
0.10
0.20
0.50
Mirror Magnetic Field B′, Gauss

Some numbers in calculations :

$$P_{n\to n'}(t) = sin^2[(\epsilon + \kappa \boldsymbol{\mu} \cdot \boldsymbol{B})t/\hbar] \approx \kappa^2(\boldsymbol{\mu} \cdot \boldsymbol{B}\ t/\hbar)^2$$

$\epsilon = \dfrac{\hbar}{\tau}$, $\tau = 10\ s \quad \to \quad \epsilon = 6.58 \times 10^{-17}\ eV$

for B=26 G and $\kappa = 10^{-6}$: $\kappa \boldsymbol{\mu} \cdot \boldsymbol{B} = 1.57 \times 10^{-16} > \epsilon$

Average velocity 720 m/s, $\mathrm{L} = 2\ \mathrm{m}$, $t = 2/720 = 2.7 \times 10^{-3} s$