

# TOWARDS *AB INITIO* COMPUTATIONS OF NEUTRINO SCATTERING ON MEDIUM-MASS NUCLEI

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in collaboration with

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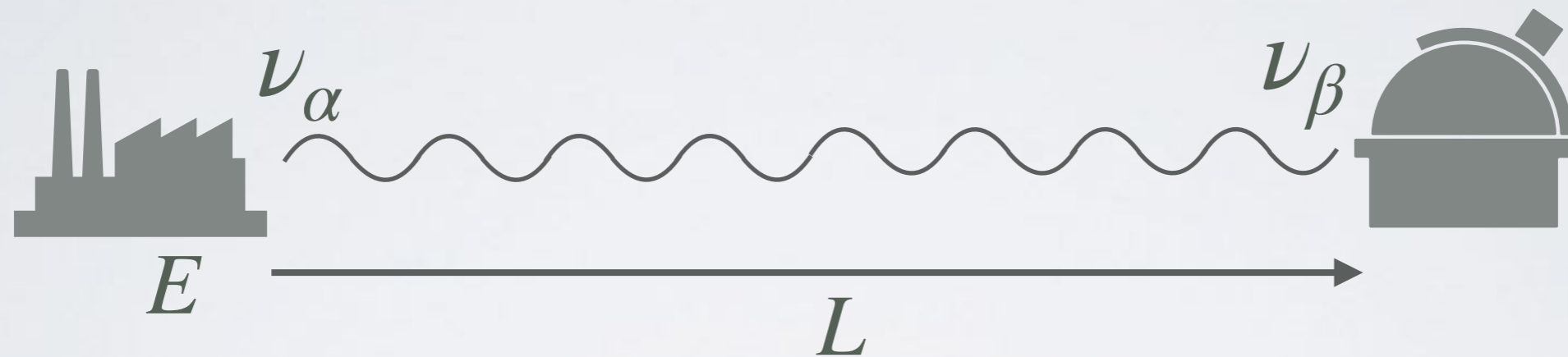
ORNL, 4 March 2021



# OUTLINE

- Motivation: neutrino oscillation experiments and their challenges
- Neutrino-nucleus scattering in MeV-GeV region
- LIT-CC approach and first results
- Outlook

# NEUTRINO OSCILLATION

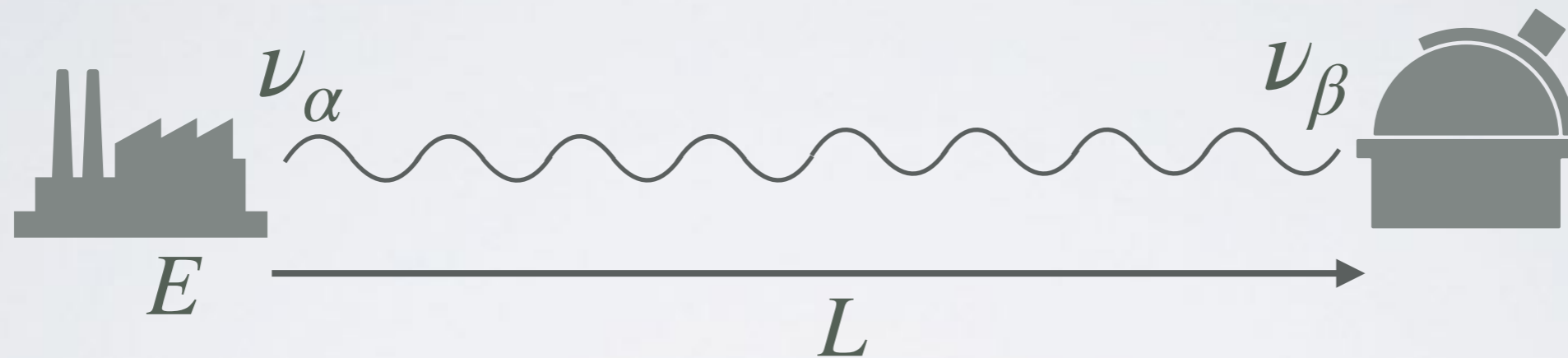


$$P(\alpha \rightarrow \beta) = \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

mixing angle (for 3 flavours we have 3 angles + CP violating phase)

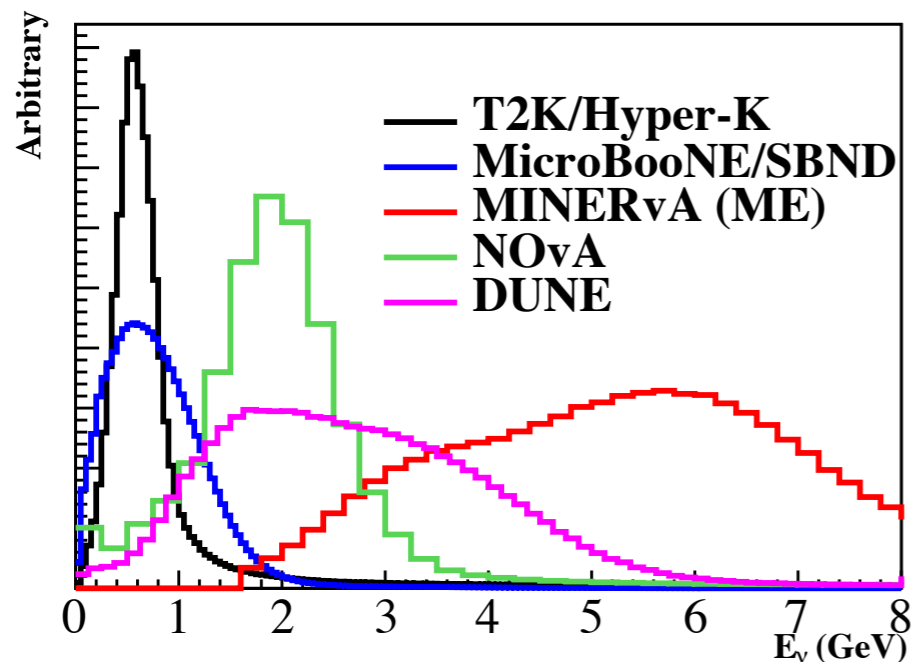
experimental setup

# NEUTRINO OSCILLATION



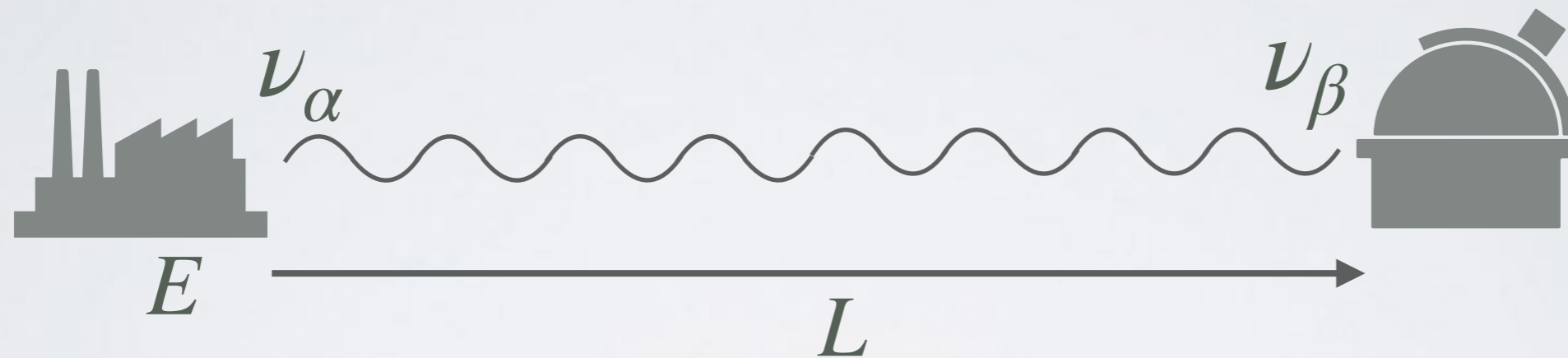
IDEA:

1. we know the initial neutrino flux (with some uncertainty)
2. we measure the rate of neutrino events in the far detector  $L$
3. we have to reconstruct neutrino energy  $E$  and flux in the far detector



From: T. Katori, M. Martini, J.Phys. G45 (2018)

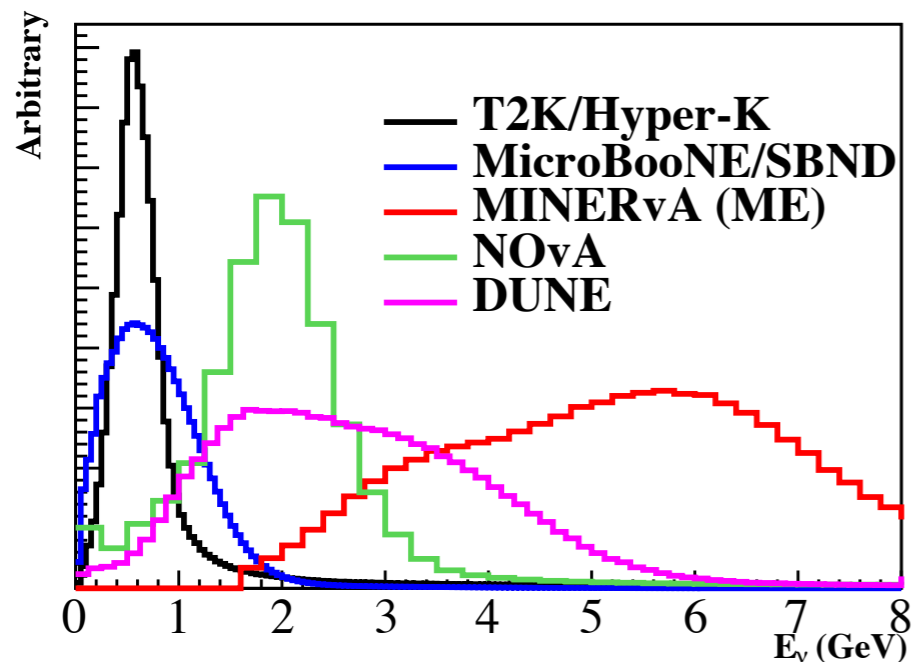
# NEUTRINO OSCILLATION



## CHALLENGES:

1. In each event  $\nu$  is not seen
2. We observe outgoing lepton
3. Depending on the detector we see some hadrons
4. Low probability of interaction

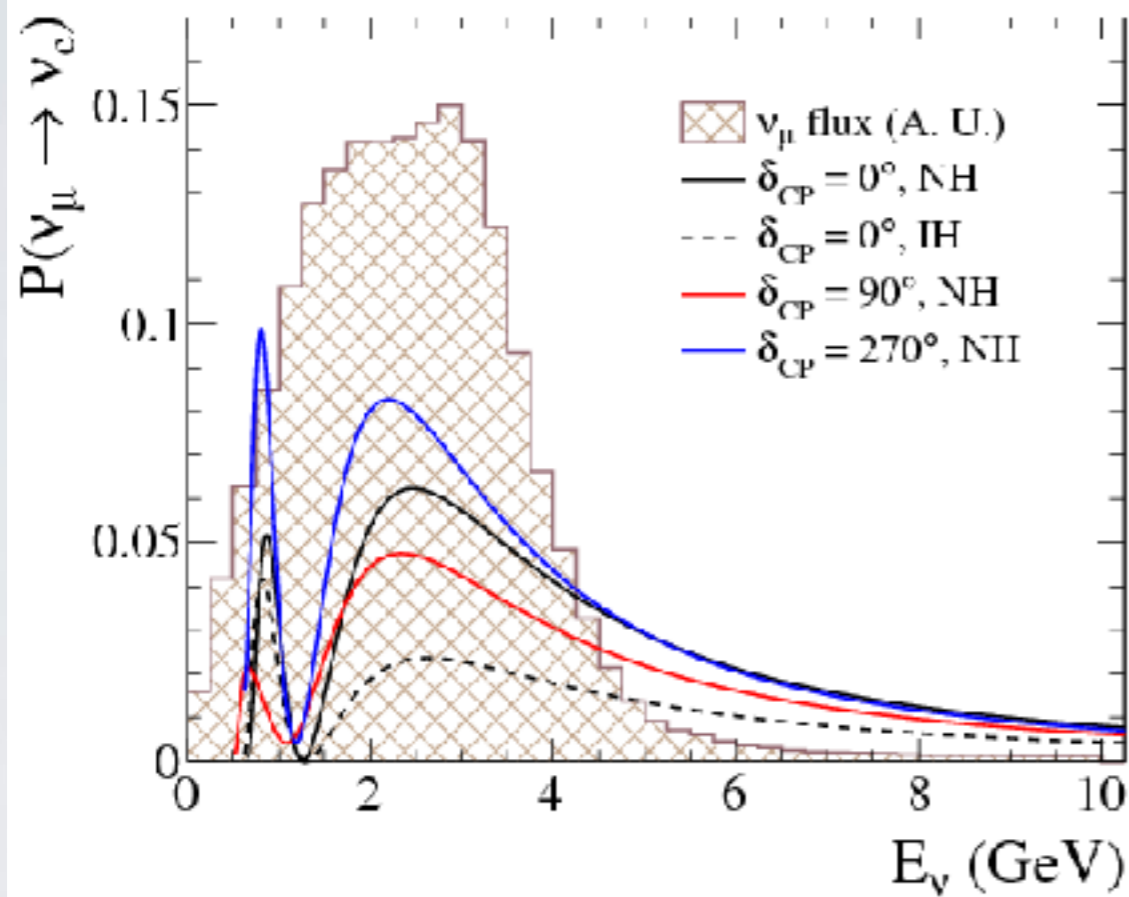
We need theoretical predictions on how neutrino interacts with nuclei



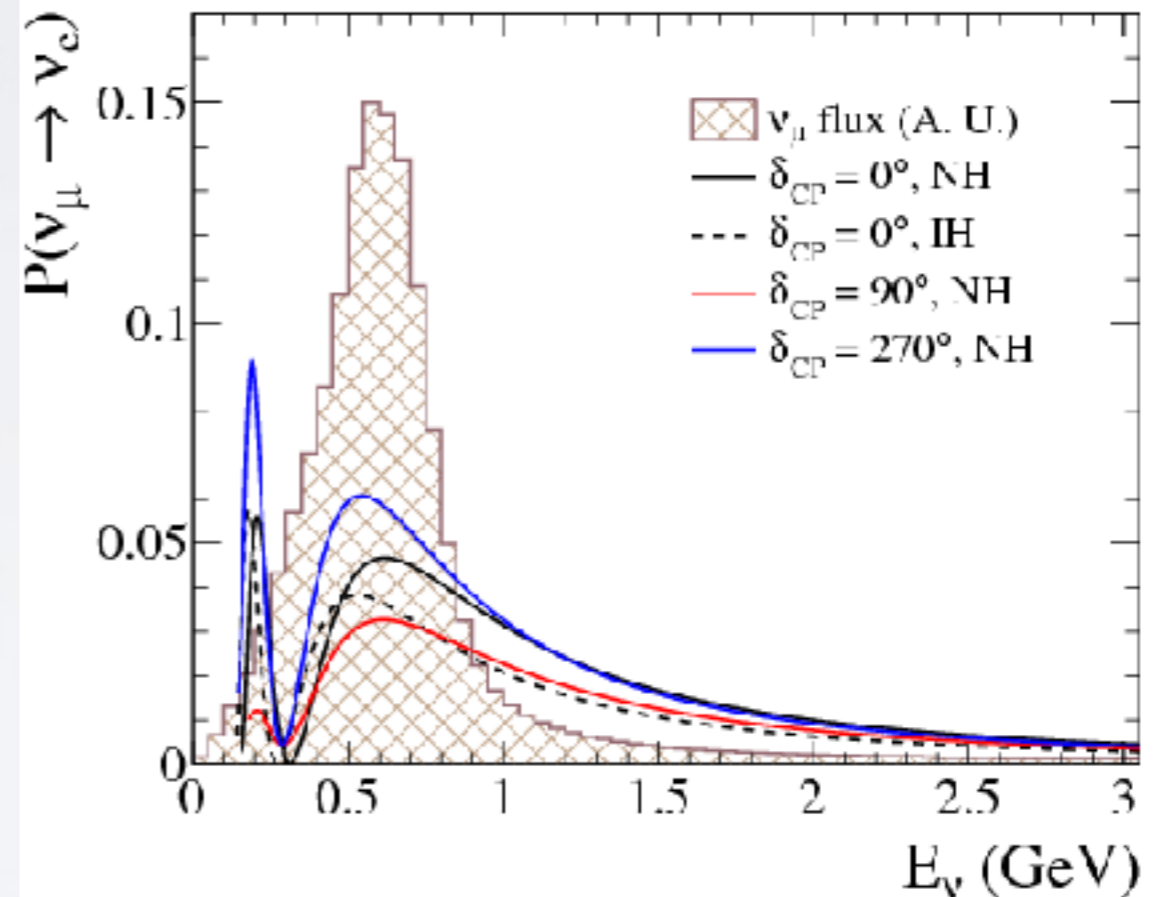
From: T. Katori, M. Martini, J.Phys. G45 (2018)

# CHALLENGES: CP VIOLATING PHASE

From: Diwan et al, Ann. Rev.Nucl. Part. Sci 66 (2016)



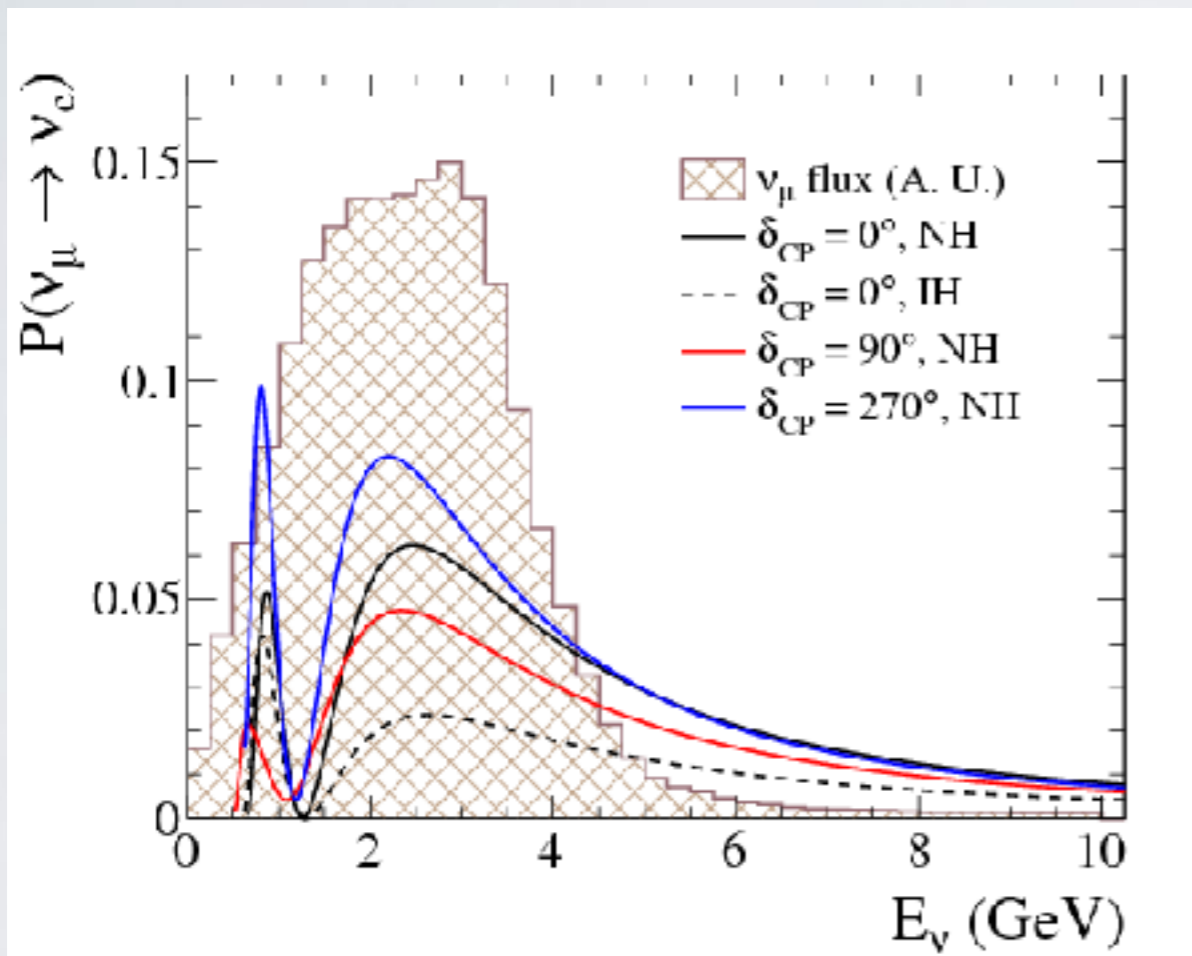
DUNE



T2HK

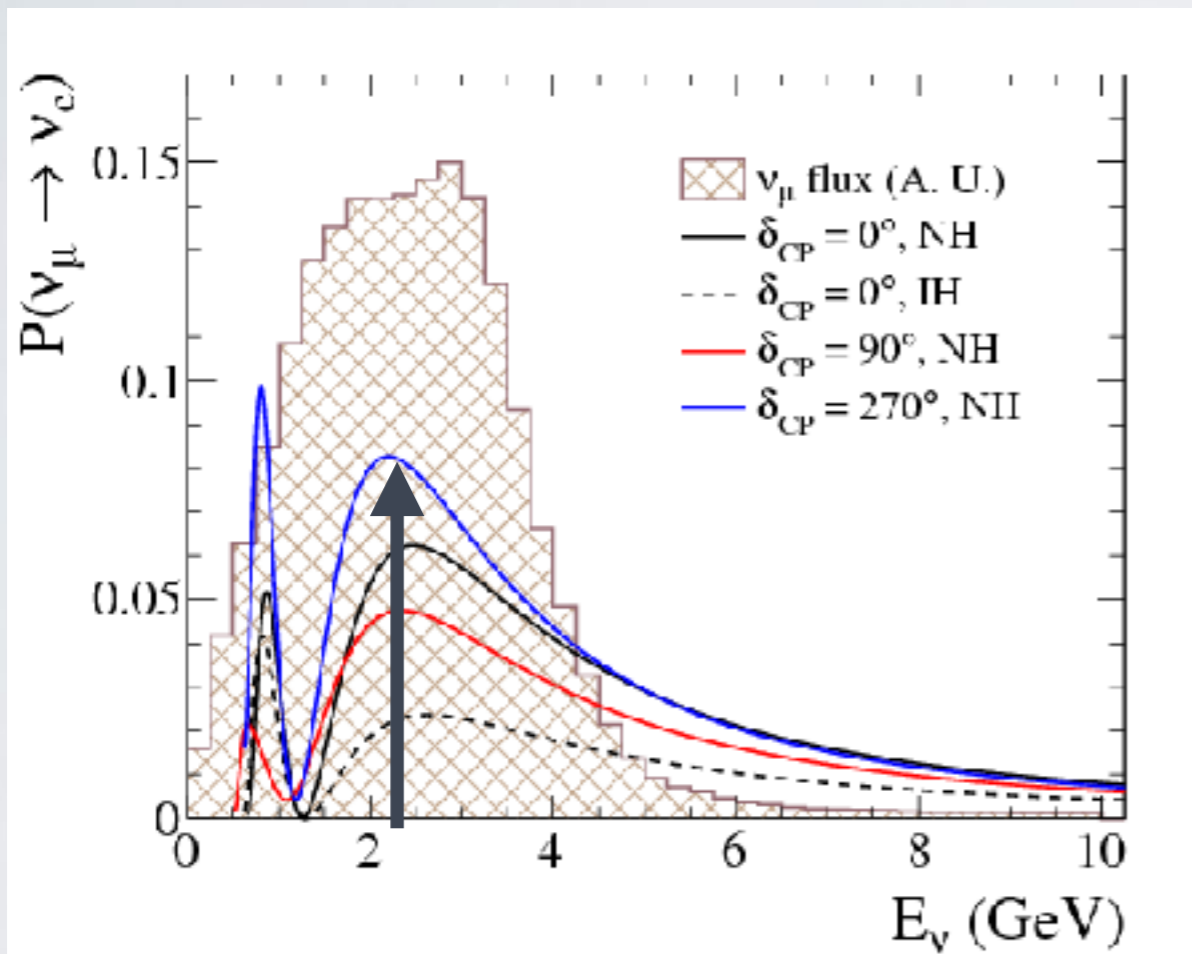
We want to reconstruct the shape of the oscillation probability curve  
(which depends on the CP phase)

# CHALLENGES



Systematic errors should be small since statistics will be high.

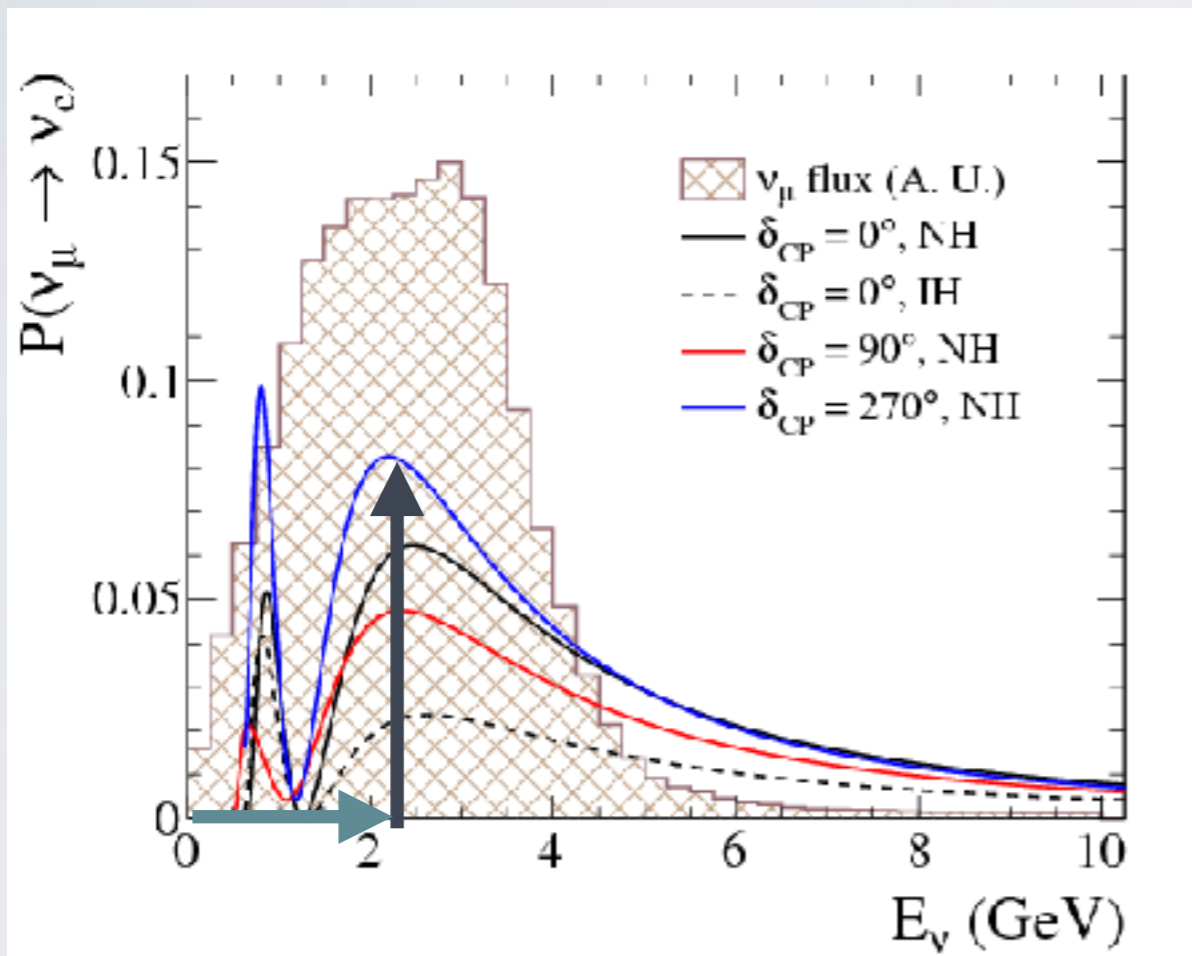
# CHALLENGES



Height of the  
oscillation peak  
(event rate)  $\propto$   
total cross section

Systematic errors should be  
small since statistics will be high.

# CHALLENGES

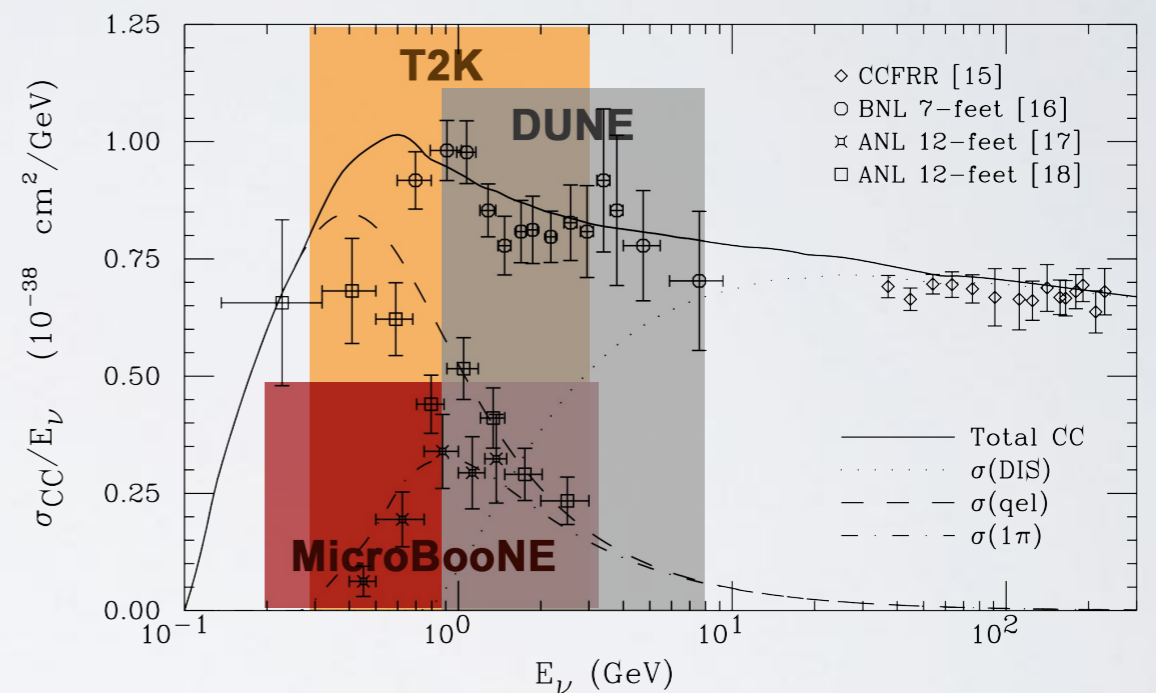
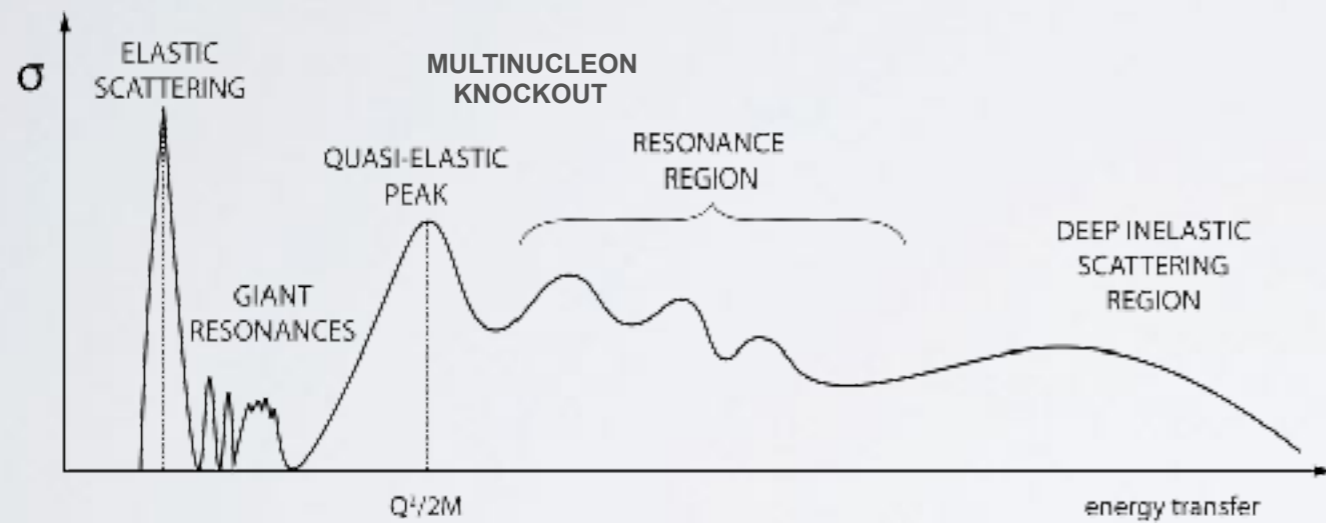


Height of the oscillation peak (event rate)  $\propto$  total cross section

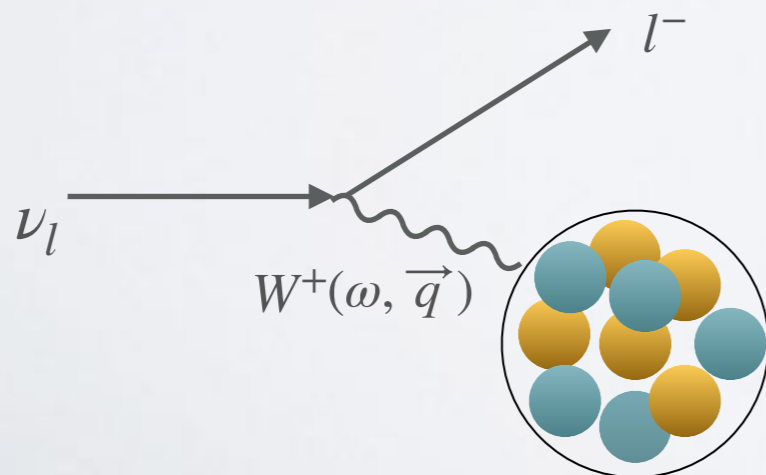
Position of the oscillation peak depends on energy reconstruction

Systematic errors should be small since statistics will be high.

# LEPTON-NUCLEUS SCATTERING



From: P. Lipari et. al., PRL 74 (1995)



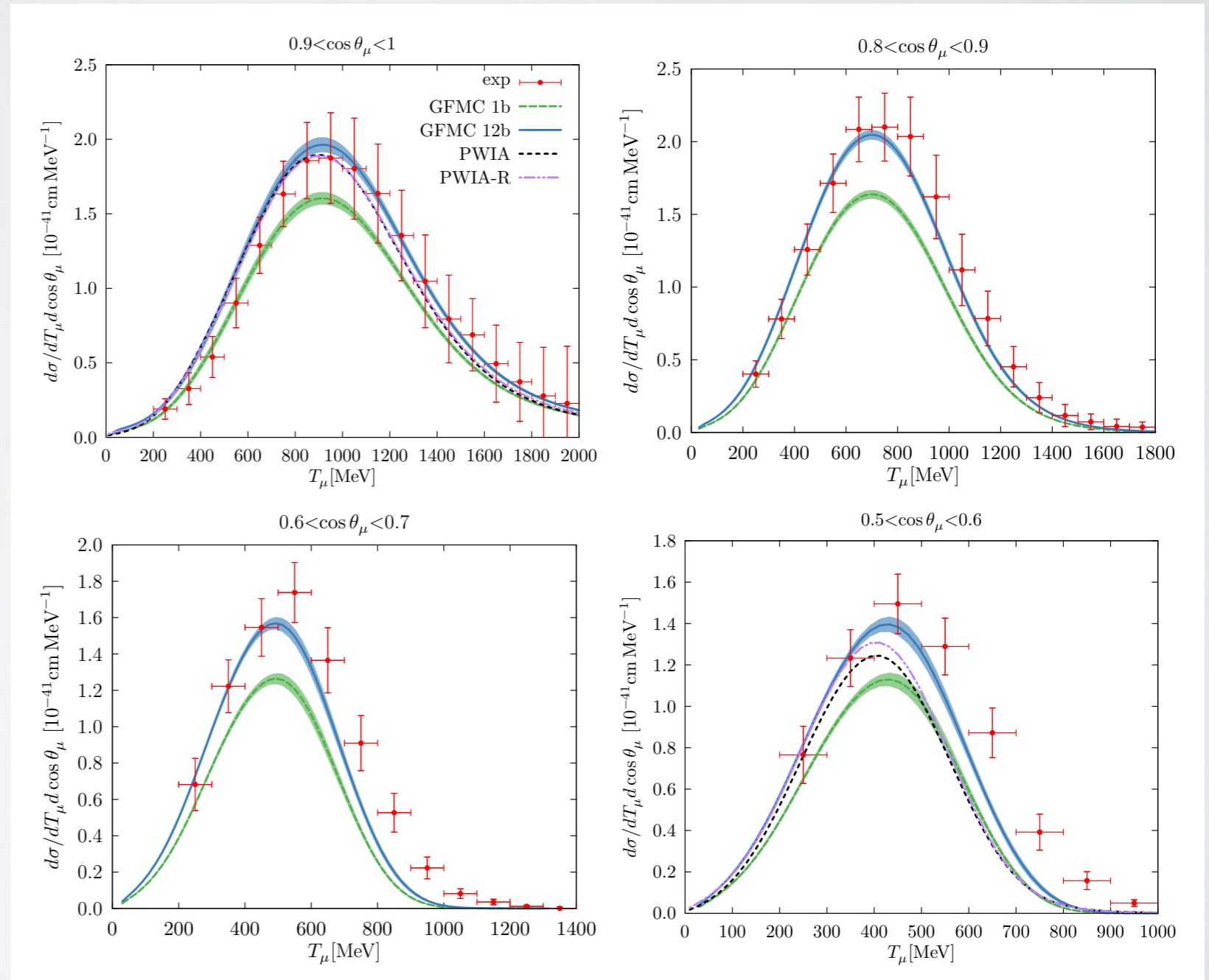
For every experiment all the physical mechanisms should be included

# NUCLEAR CROSS-SECTIONS

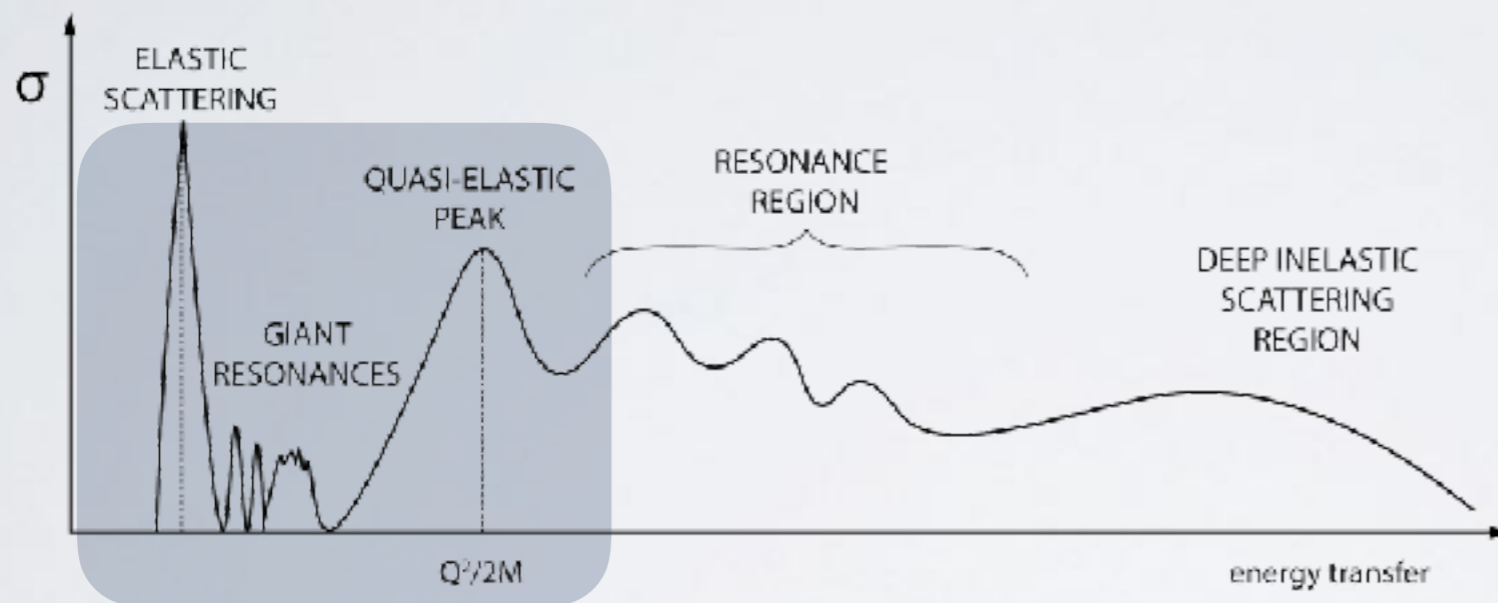
- ✓ Theoretical models enter analysis through Monte Carlo generators
- ✓ Simpler models should be benchmarked with more reliable methods at the level of inclusive cross-section
- ✓ Important targets:  $^{12}\text{C}$ ,  $^{16}\text{O}$  (near&far detector T2HK),  $^{40}\text{Ar}$  (DUNE)

# AB INITIO RESULTS FOR $^{12}\text{C}$

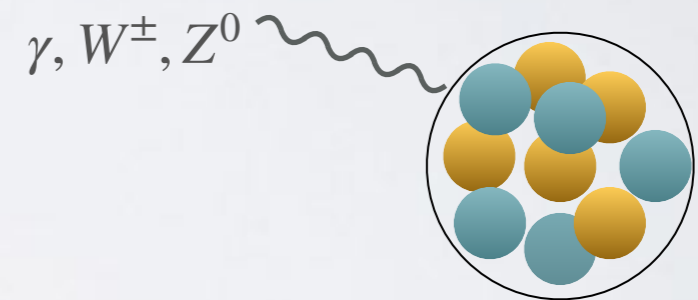
- First GFMC comparison with experimental neutrino data
- Important contribution of 2-body currents
- GFMC suitable for light nuclei  $A \leq 12$



# NUCLEAR RESPONSE



$$J_\mu = (\rho, \vec{j}) | \Psi \rangle$$



$$\sigma \propto L^{\mu\nu} R_{\mu\nu}$$

lepton tensor nuclear responses

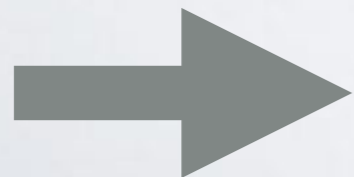
$$R_{\mu\nu}(\omega, q) = \sum_f \langle \Psi | J_\mu^\dagger(q) | \Psi_f \rangle \langle \Psi_f | J_\nu(q) | \Psi \rangle \delta(E_0 + \omega - E_f)$$

# ELECTRONS FOR NEUTRINOS

$$\left. \frac{d\sigma}{d\omega dq} \right|_{\nu/\bar{\nu}} = \sigma_0 \left( v_{CC} R_{CC} + v_{CL} R_{CL} + v_{LL} R_{LL} + v_T R_T \pm v_{T'} R_{T'} \right)$$

$$\left. \frac{d\sigma}{d\omega dq} \right|_e = \sigma_M \left( v_L R_L + v_T R_T \right)$$

- ✓ much more precise data
- ✓ we can get access to  $R_L$  and  $R_T$  separately (Rosenbluth separation)
- ✓ experimental programs of electron scattering in JLab, MESA



We will start with longitudinal response

# AB INITIO NUCLEAR THEORY FOR NEUTRINOS

✓ Nuclear Hamiltonian

$$\mathcal{H} |\Psi\rangle = E |\Psi\rangle$$

✓ Electroweak currents

$$J^\mu = (\rho, \vec{j})$$

✓ Many-body method

$$\mathcal{A} = \langle \Psi_m | J_\mu | \Psi_n \rangle$$

# NUCLEAR HAMILTONIAN

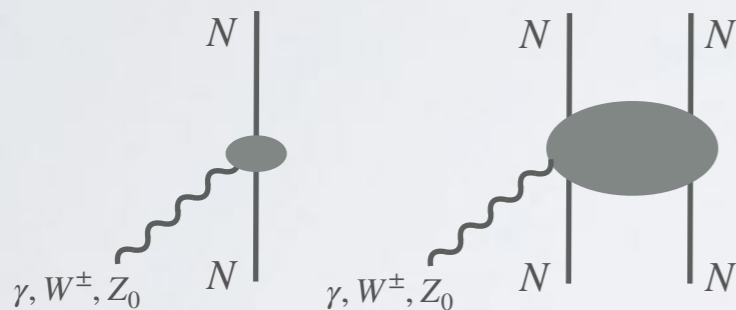
$$\mathcal{H} = \sum_i \frac{p_i^2}{2m} + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

		2N force	3N force	4N force
$n = 0$	LO			
$n = 2$	NLO			
$n = 3$	N2LO			
$n = 4$	N3LO			

- Chiral Hamiltonians exploiting chiral symmetry (QCD);  $\pi$ ,  $N$ , ( $\Delta$ ) degrees of freedom
- counting scheme in  $\left(\frac{Q}{\Lambda}\right)^n$
- low energy constants (LEC) fit to data
- uncertainty assessment

# ELECTROWEAK CURRENTS

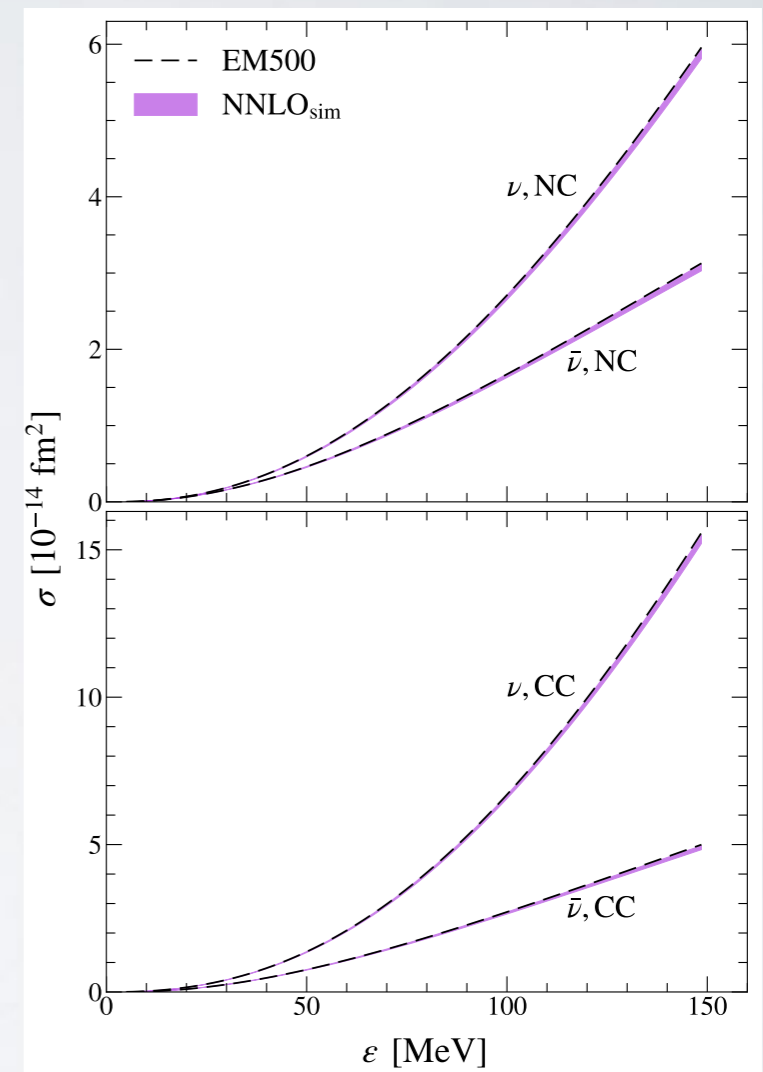
$$J = \sum_i J_i + \sum_{i < j} J_{ij} + \dots$$



known to give significant contribution for neutrino-nucleus scattering

Current decomposition into multipoles needed for various *ab initio* methods: CC, No Core Shell Model, In-Medium Similarity Renormalization Group

$$\nu(\bar{\nu}) + d \rightarrow \mu^\pm + X$$



Multipole decomposition for 1- and 2-body EW currents

# COUPLED CLUSTER METHOD

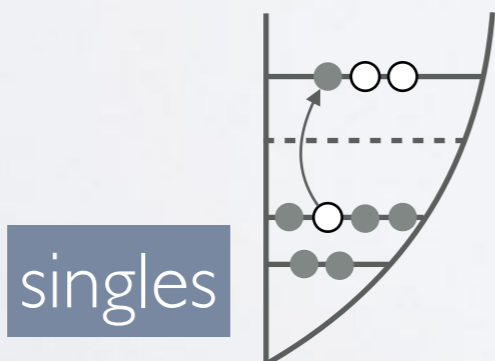
Reference state (Hartree-Fock):  $|\Psi\rangle$

Include correlations through  $e^T$  operator

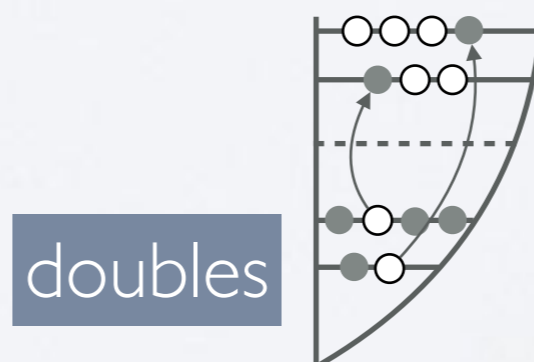
similarity transformed  
Hamiltonian (non-Hermitian)

$$e^{-T} \mathcal{H} e^T |\Psi\rangle \equiv \bar{\mathcal{H}} |\Psi\rangle = E |\Psi\rangle$$

Expansion:  $T = \sum t_a^i a_a^\dagger a_i + \sum t_{ab}^{ij} a_a^\dagger a_b^\dagger a_i a_j + \dots$



singles



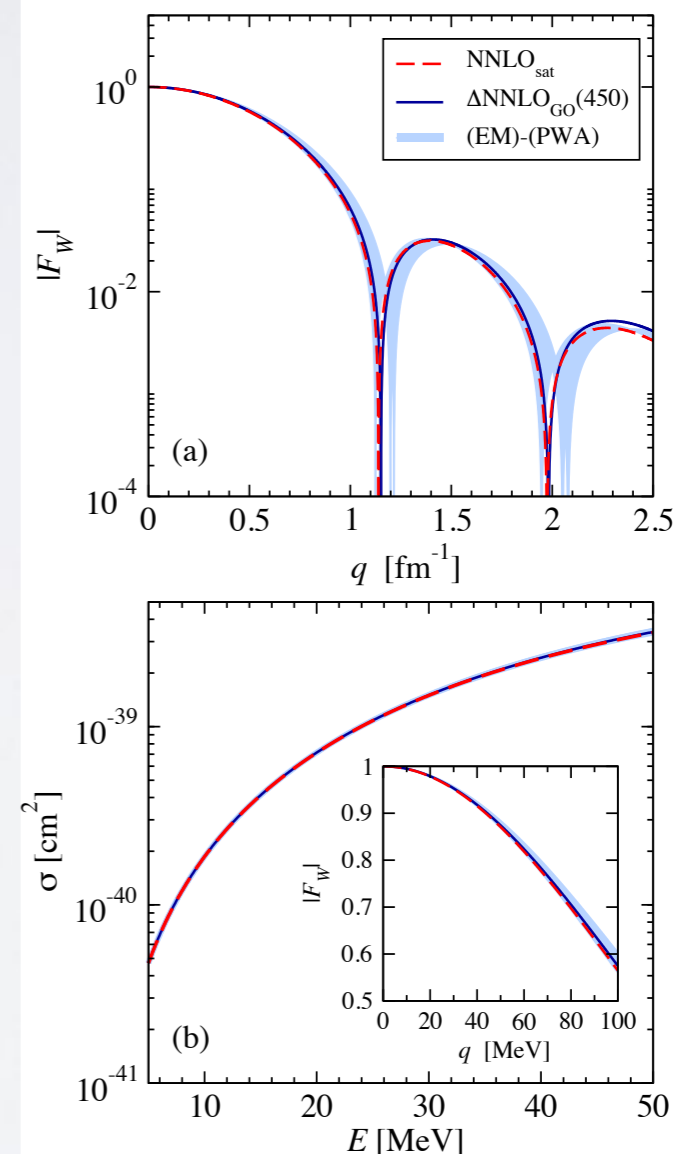
doubles

← coefficients obtained  
through coupled cluster  
equations

# COUPLED CLUSTER METHOD

- ✓ Controlled approximation through truncation in  $T$
- ✓ Polynomial scaling with  $A$  (predictions for  $^{100}\text{Sn}$ )
- ✓ Size extensive
- ✓ Works most efficiently for doubly magic nuclei

coherent elastic neutrino scattering on  $^{40}\text{Ar}$



# LORENTZ INTEGRAL TRANSFORM

$$R_{\mu\nu}(\omega, q) = \int_f \langle \Psi | J_\mu^\dagger | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi \rangle \delta(E_0 + \omega - E_f)$$

continuum spectrum

Instead we calculate

$$S_{\mu\nu}(\omega, q) = \int d\sigma K(\omega, \sigma) R_{\mu\nu}(\omega, q) = \int d\sigma \langle \Psi | J_\mu^\dagger K(\mathcal{H} - E_0, \sigma) J_\nu | \Psi \rangle$$

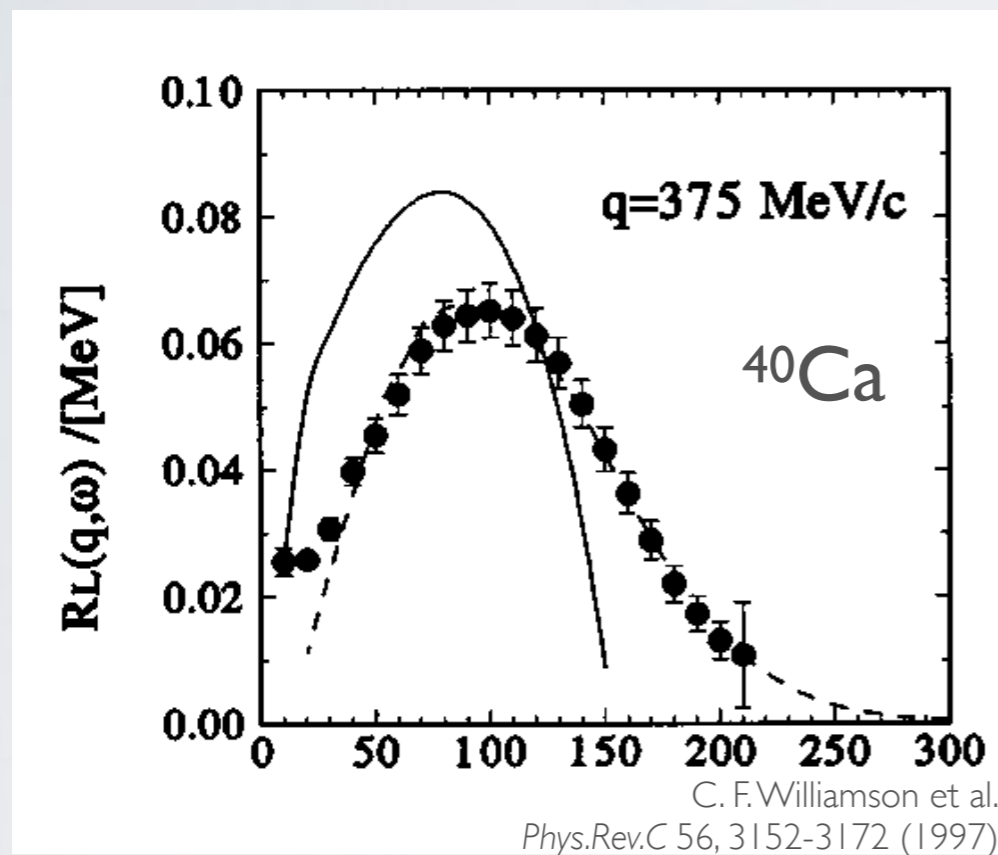
$S_{\mu\nu}$  has to be inverted to get access to  $R_{\mu\nu}$

Lorentzian kernel:

$$K_\Lambda(\omega, \sigma) = \frac{1}{\pi} \frac{\Lambda}{\Lambda^2 + (\omega - \sigma)^2}$$

→ LIT-CC used for photo-absorption

# LONGITUDINAL RESPONSE AND COULOMB SUM RULE

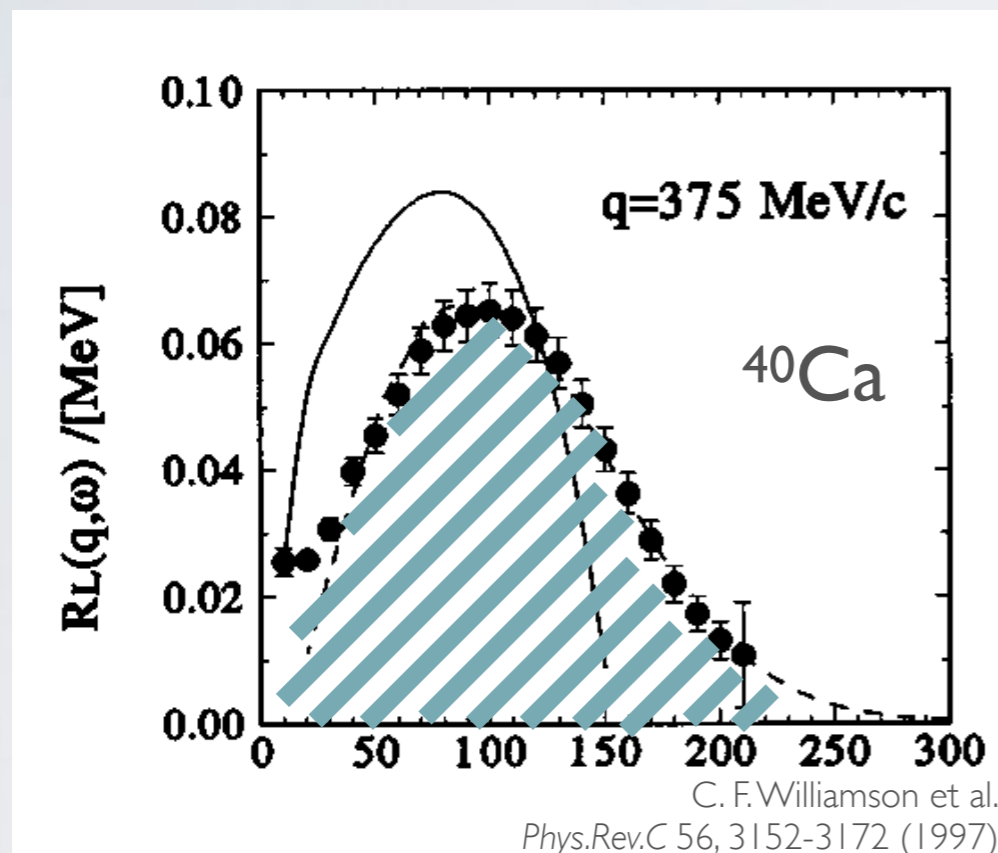


$$\text{charge operator } \hat{\rho}(q) = \sum_{j=1}^Z e^{iqz'_j}$$

- ➔ operator multipole decomposition (and sum)
- ➔ higher energy-momentum transfer than considered earlier
- ➔ translationally non-invariant operator

$$R_L(\omega, q) = \sum_f \frac{\langle \Psi | \hat{\rho}^\dagger(q) | \Psi_f \rangle \langle \Psi_f | \hat{\rho}(q) | \Psi \rangle \delta(E_0 + \omega - E_f)}{\omega}$$

# LONGITUDINAL RESPONSE AND COULOMB SUM RULE

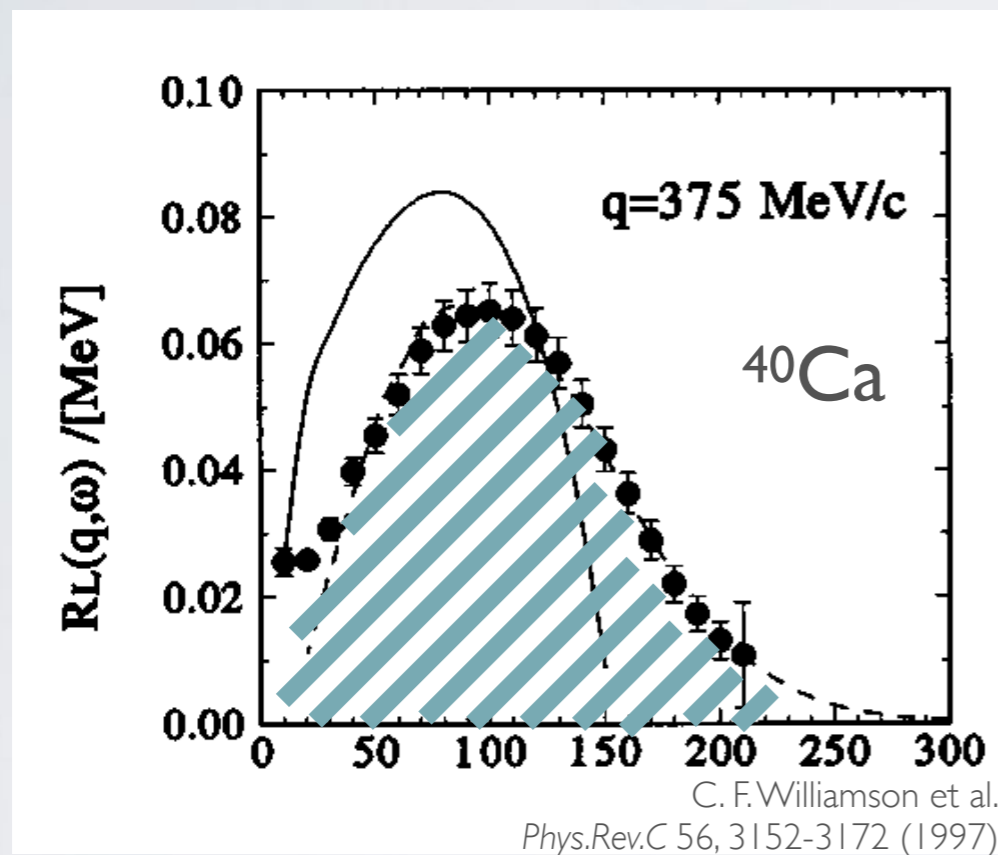


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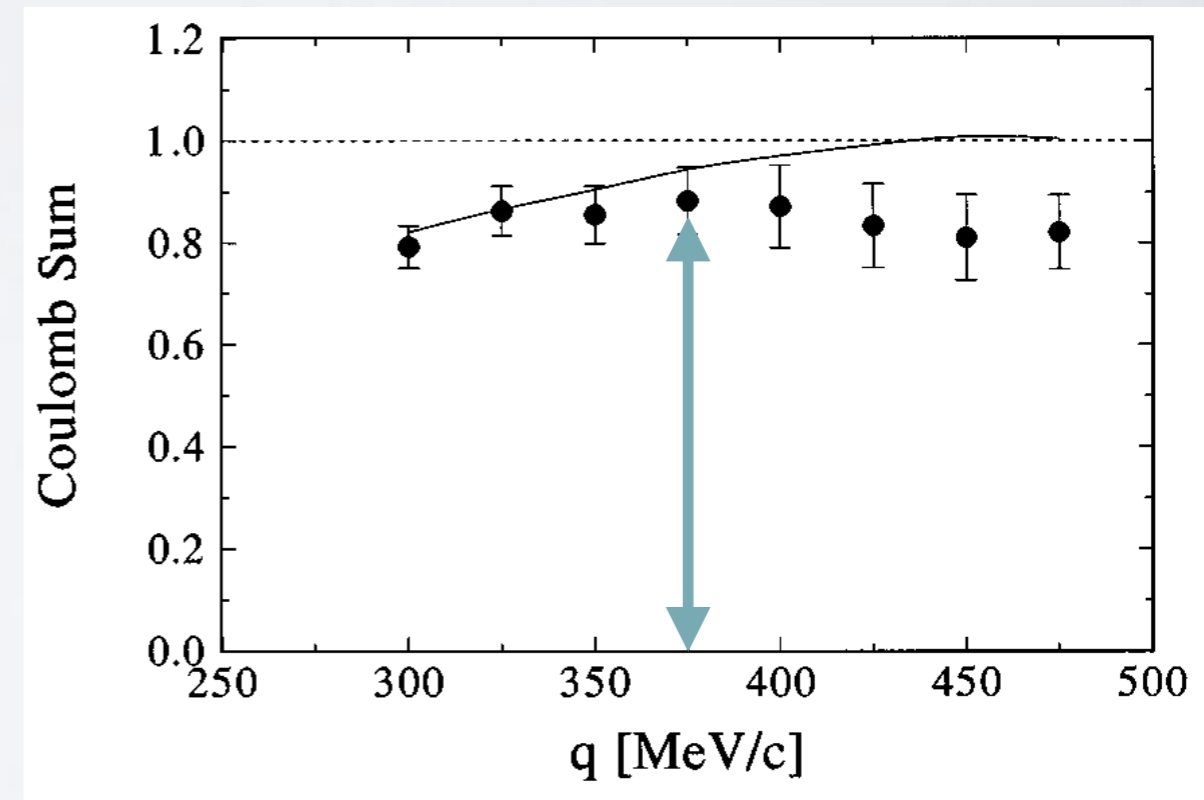
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$$m_0(q) = \int d\omega R_L(\omega, q) = \sum_{f \neq 0} |\langle \Psi_f | \hat{\rho} | \Psi \rangle|^2$$

# COULOMB SUM RULE

$$m_0(q) = \int d\omega R_L(\omega, q) = \sum_{f \neq 0} |\langle \Psi_f | \hat{\rho} | \Psi \rangle|^2 = \langle \Psi | \hat{\rho}^\dagger \hat{\rho} | \Psi \rangle - |F_{el}(q)|^2$$

easier to calculate since  
we do not need  $|\Psi_f\rangle$

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center of mass problem

$|\Psi\rangle$  has  $3A$  coordinates  $\rightarrow$   $3(A-1)$  intrinsic coordinates +  $\vec{R} = \frac{1}{A} \sum_i^A \vec{r}_i$

With translationally non-invariant operators we may excite spurious states

# COULOMB SUM RULE

Project out spurious states:  $\hat{\rho} |\Psi\rangle = |\Psi_{phys}\rangle + |\Psi_{spur}\rangle$

It has been shown that to good approximation the ground state factorizes:

$$|\Psi\rangle = |\Psi_I\rangle |\Psi_{CoM}\rangle$$

center of mass wave function is a Gaussian

G. Hagen, T. Papenbrock, D. Dean  
*Phys.Rev.Lett.* 103 (2009) 062503

We follow a similar ansatz for the excited states:

$$\hat{\rho} |\Psi\rangle = |\Psi_I^{exc}\rangle |\Psi_{CoM}\rangle + |\Psi_I\rangle |\Psi_{CoM}^{exc}\rangle$$

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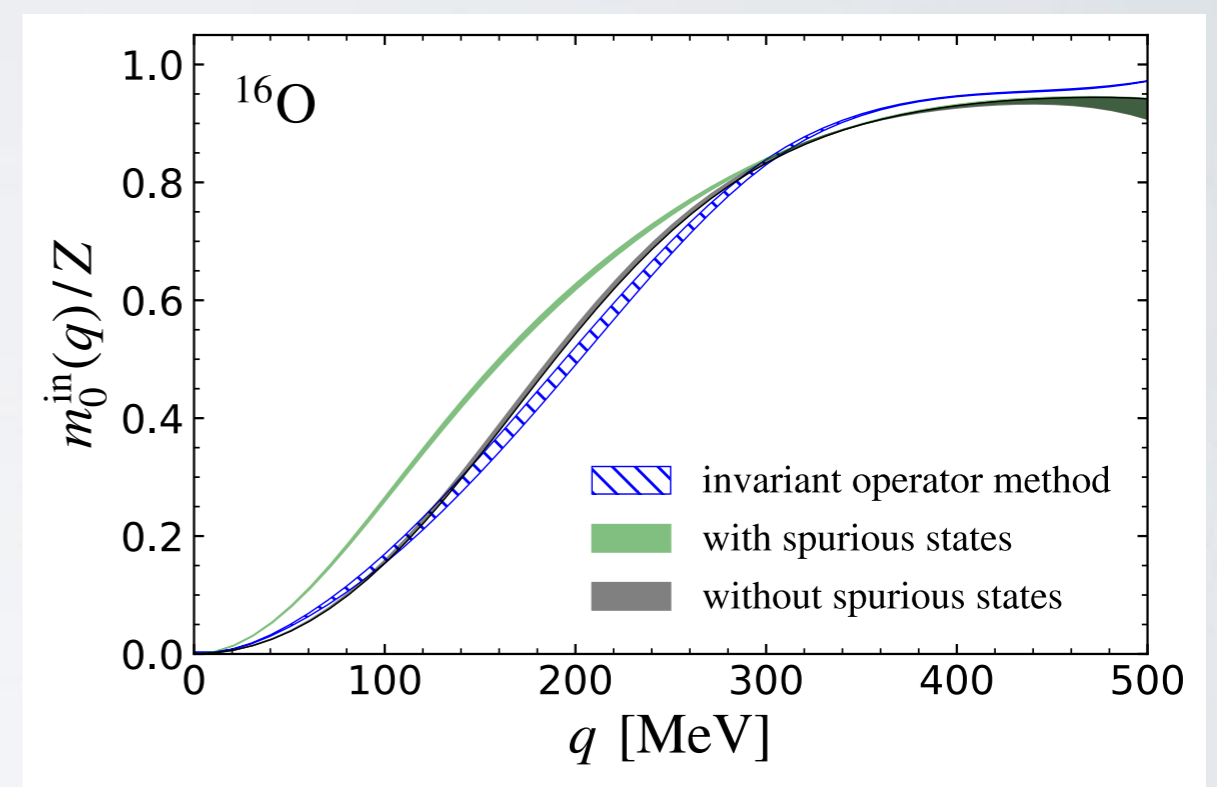
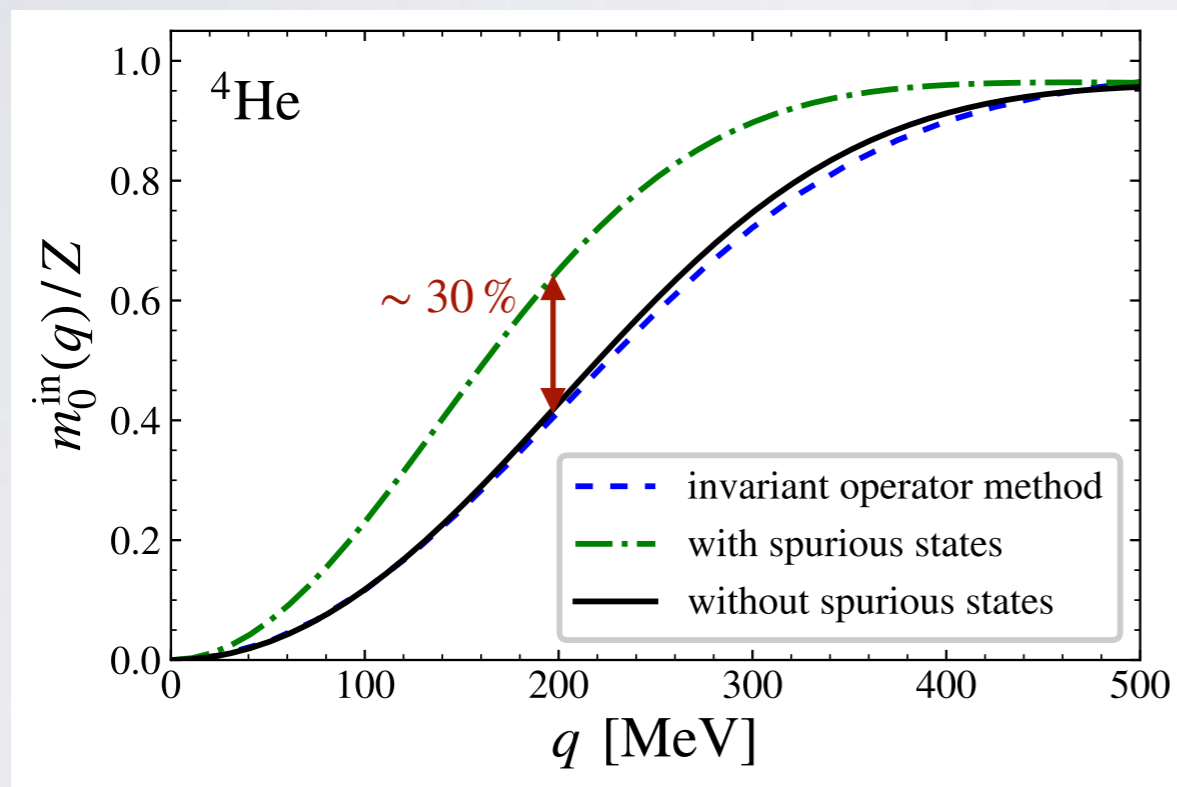
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spurious

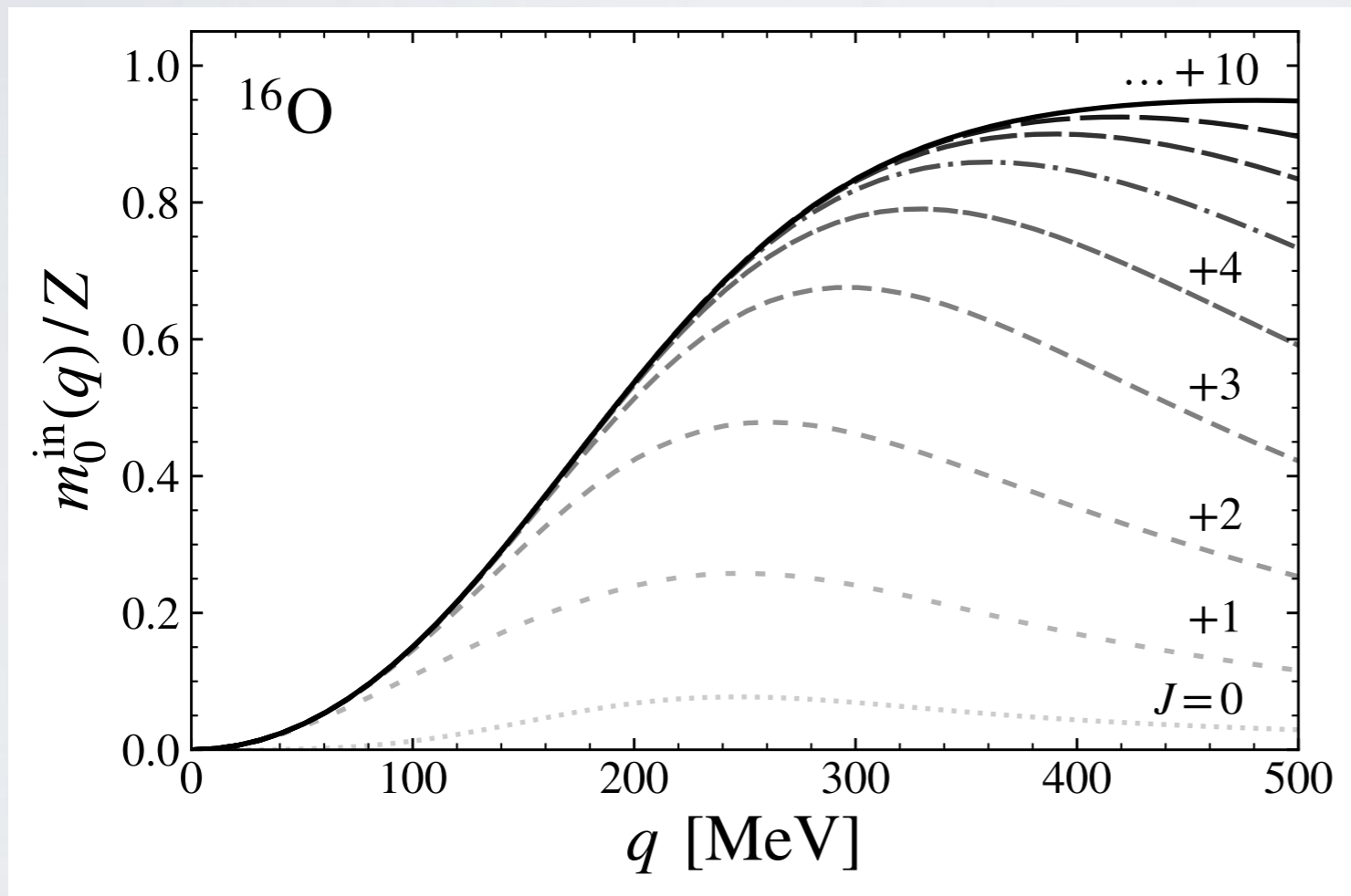
# COULOMB SUM RULE



J.E.S. B. Acharya, S. Bacca, G. Hagen  
*Phys. Rev. C* 102 (2020) 064312

CoM spurious states dominate for light nuclei

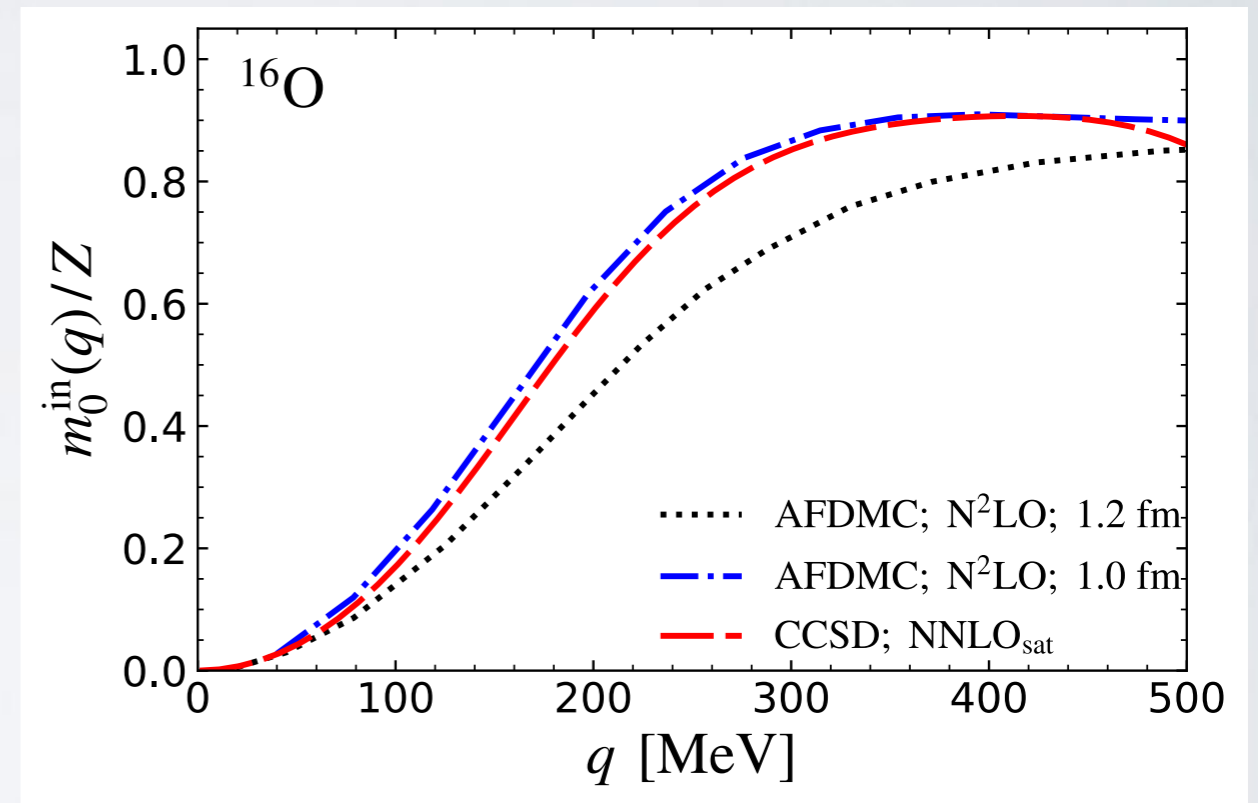
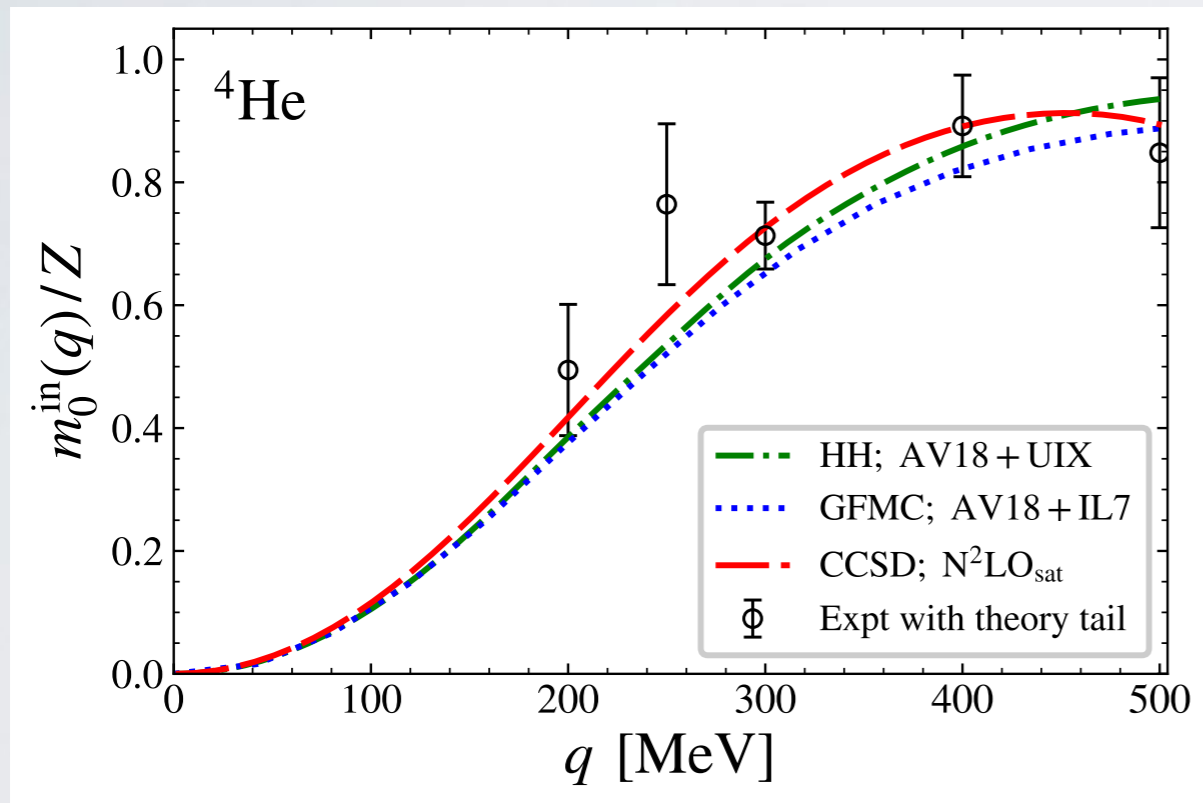
# COULOMB SUM RULE



Multipole sum

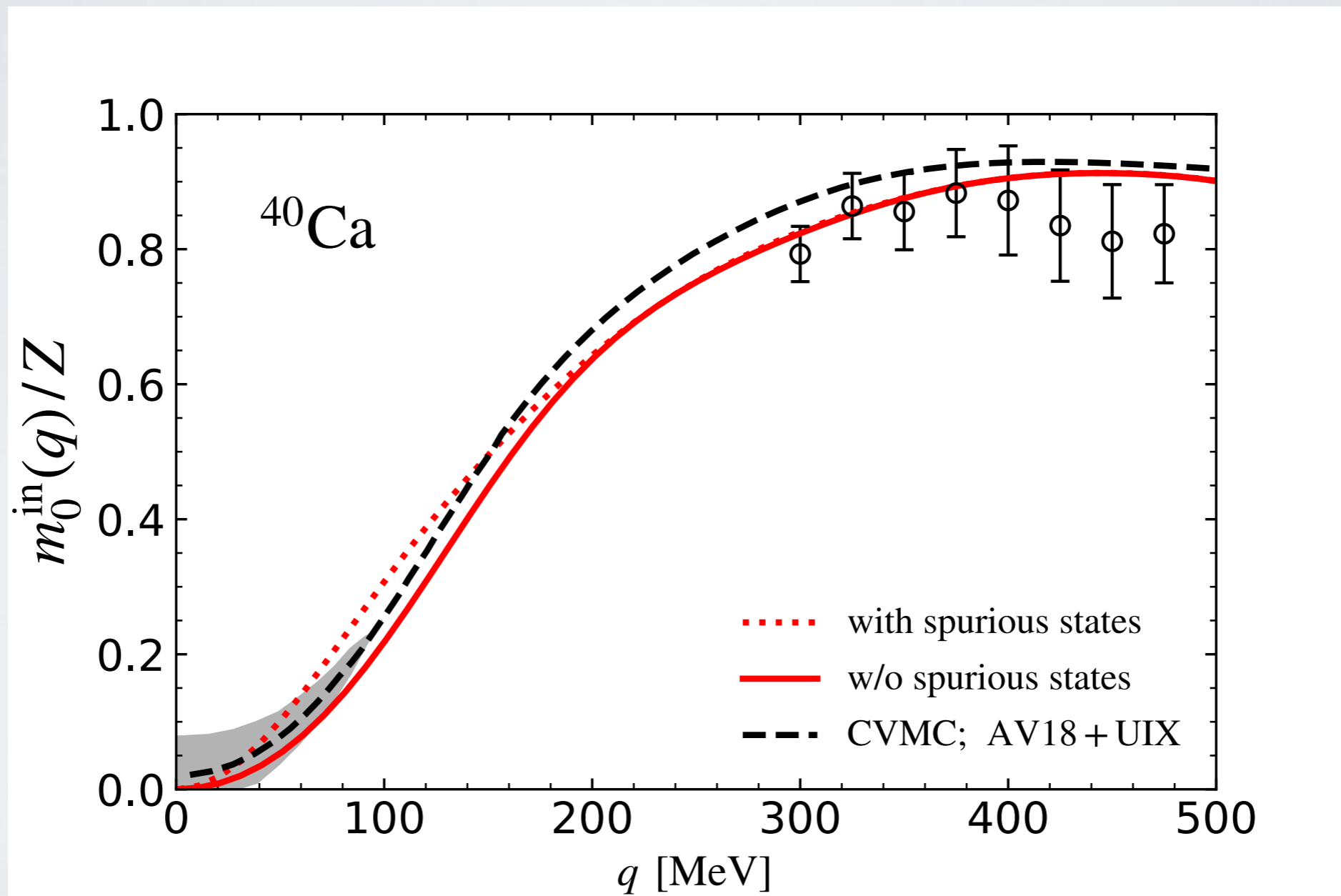
$$\hat{\rho} = \sum_{J=0}^{\infty} [\hat{\rho}]^J$$

# COULOMB SUM RULE



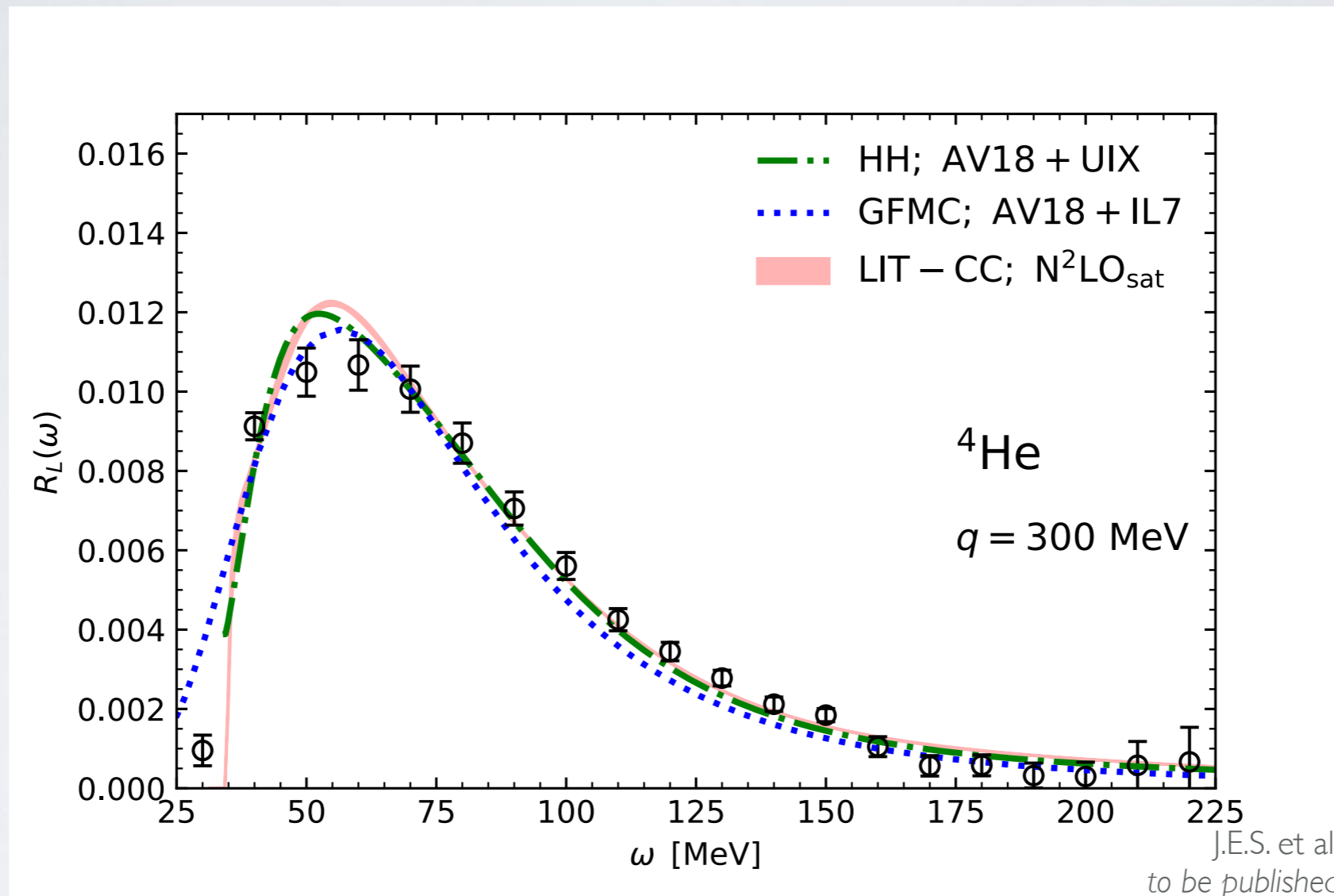
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# COULOMB SUM RULE



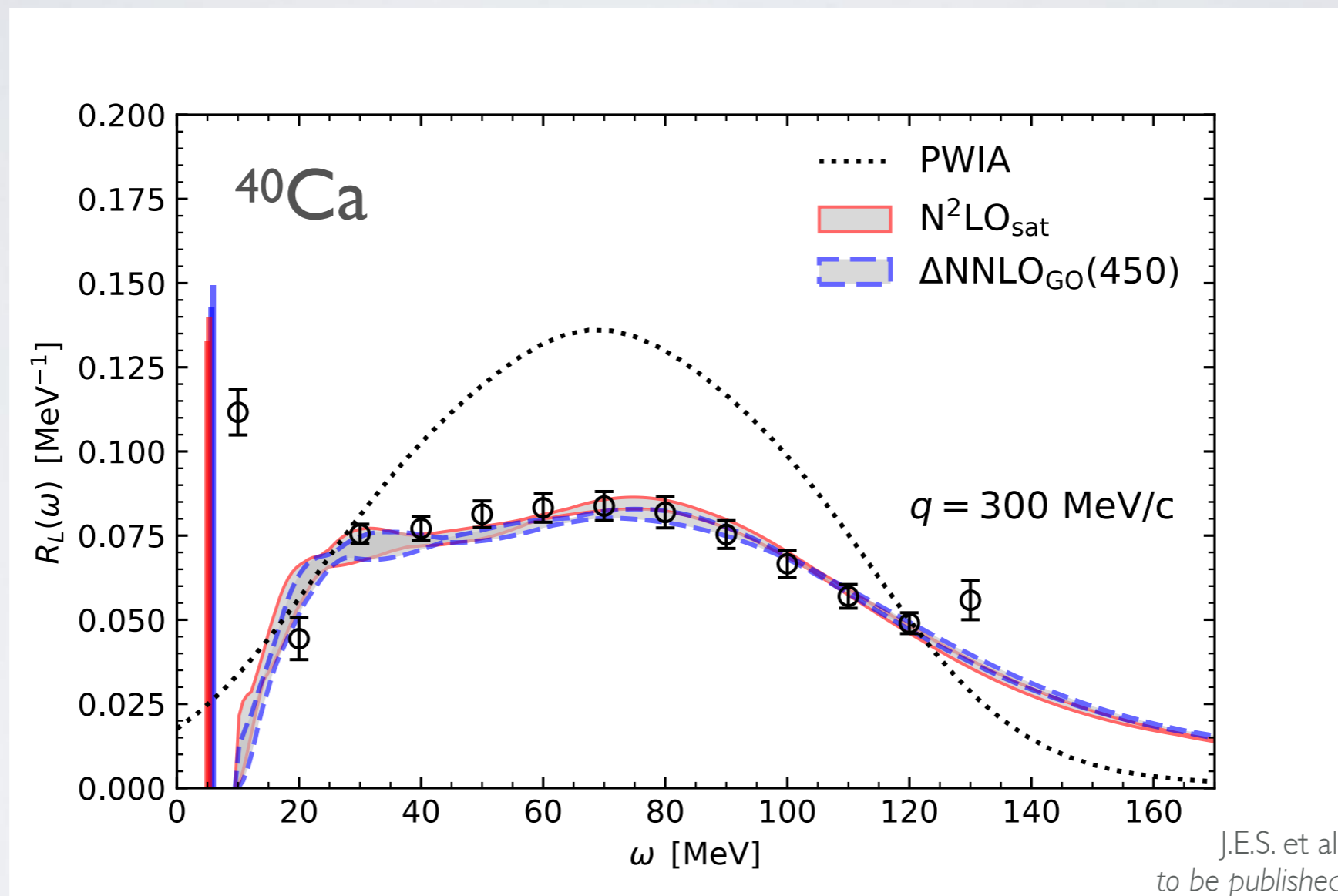
J.E.S. et al.  
to be published

# LONGITUDINAL RESPONSE



Uncertainty band: inversion procedure

# LONGITUDINAL RESPONSE



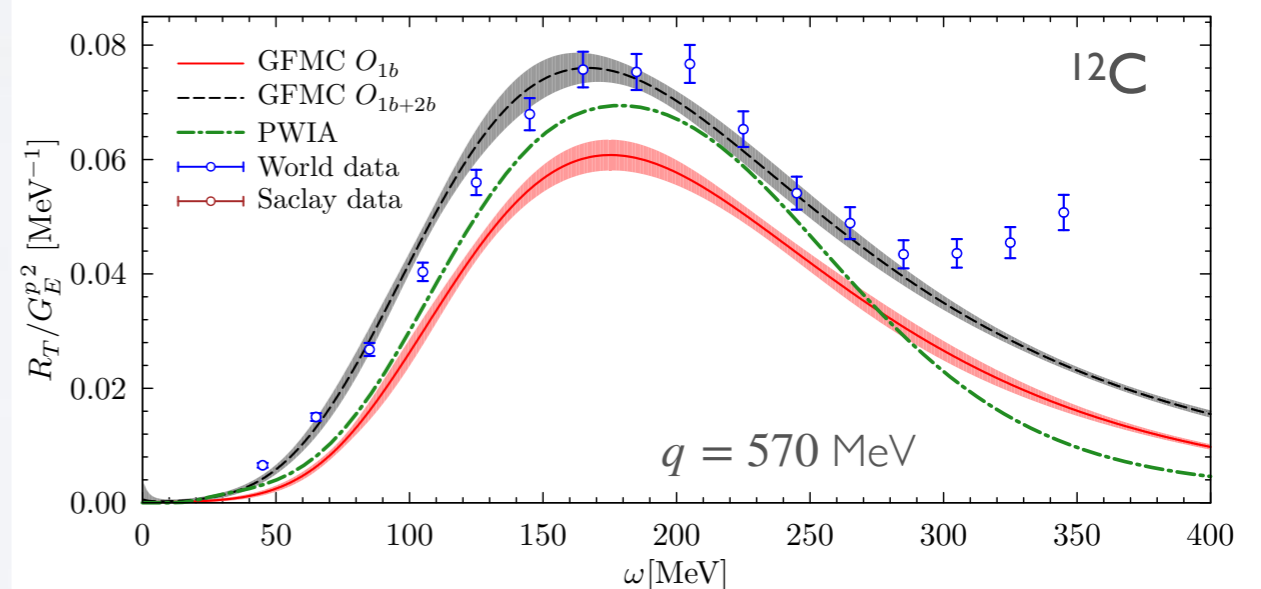
- Results for 2 different chiral potentials
- Comparison with Plane wave impulse approximation (PWIA)

# OUTLOOK

$$\left. \frac{d\sigma}{d\omega dq} \right|_e = \sigma_M \left( v_L R_L + v_T R_T \right)$$

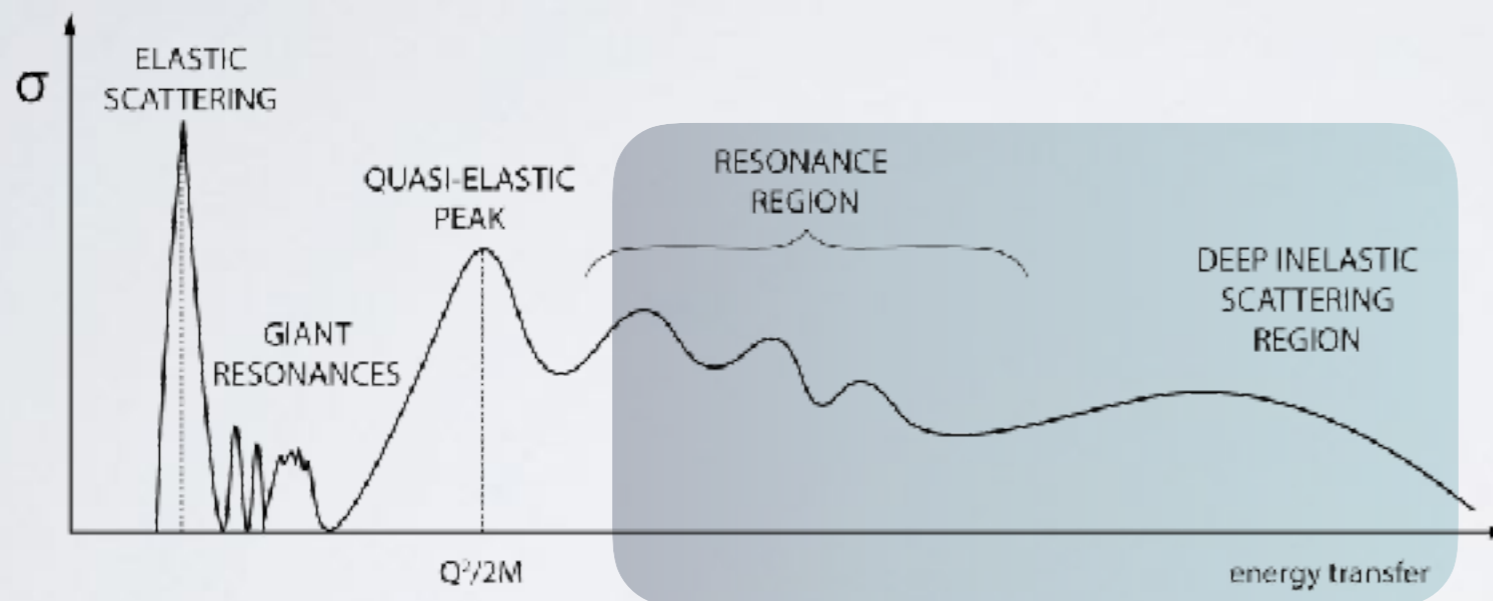
comparison with electron scattering cross-sections on  ${}^4\text{He}$ ,  ${}^{16}\text{O}$ ,  ${}^{40}\text{Ca}$

Transverse response (where the 2-body currents are relevant)



A. Lovato et al.  
*Phys.Rev.Lett.* 117 (2016) 8, 082501

# WHAT ELSE CAN BE DONE WITH AB INITIO THEORY?



How to account for the nuclear effects at higher energies?

Impulse Approximation  
is valid when  $q$  is high enough (resolution+final state interaction negligible).

Spectral function  
Probability distribution of finding a nucleon of a given momentum and energy in the nucleus.

# CONCLUSIONS

- Nuclear physics is challenged by neutrino oscillation experiments
- Setting stage for neutrino-nucleus cross-section calculation:
  - 2-body currents decomposition into multipoles
  - removal of spurious states
  - application of LIT-CC method for momentum transfer  $q \leq 500$  MeV
- With LIT-CC we obtained first ab initio results for longitudinal response for medium-size nuclei ( $^{16}\text{O}$  and  $^{40}\text{Ar}$  can be addressed)

THANK YOU