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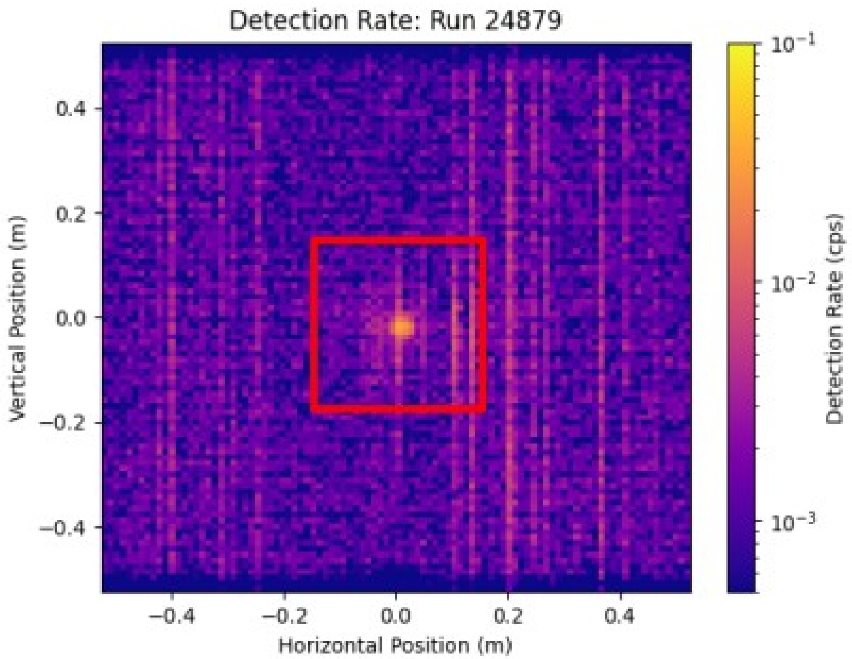
Analyzing GPSANS data from 2021 Experiment

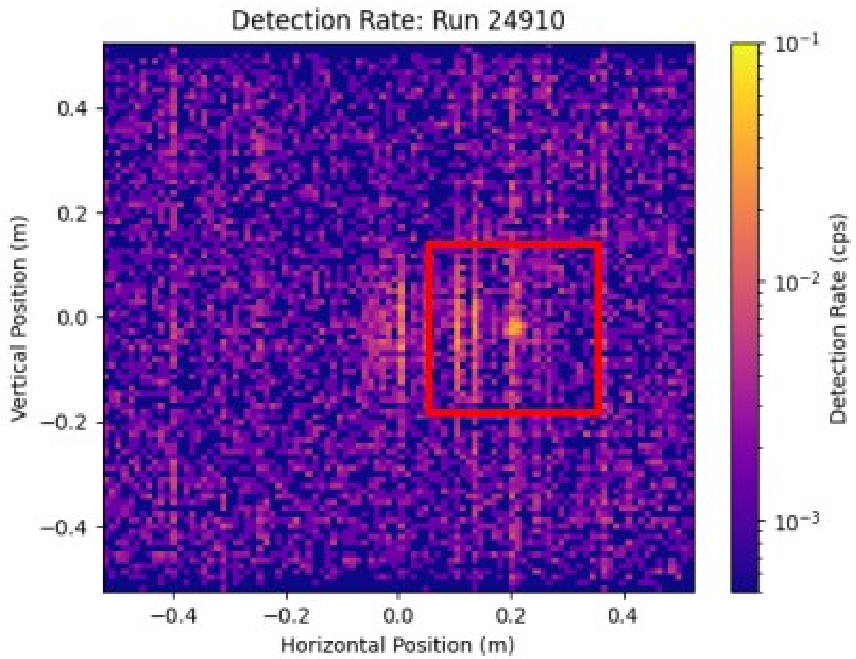
Estimating fit error using multivariate parameter sampling, analyzing statistical effect of |B| on neutron oscillation

Region of Interest Definition

Size and position of roi defined by centering focus and finding square of size which contains 95% of detections of attenuated beam (24x24cm)

- Detector position shifted ~20cm between runs 24902-24904

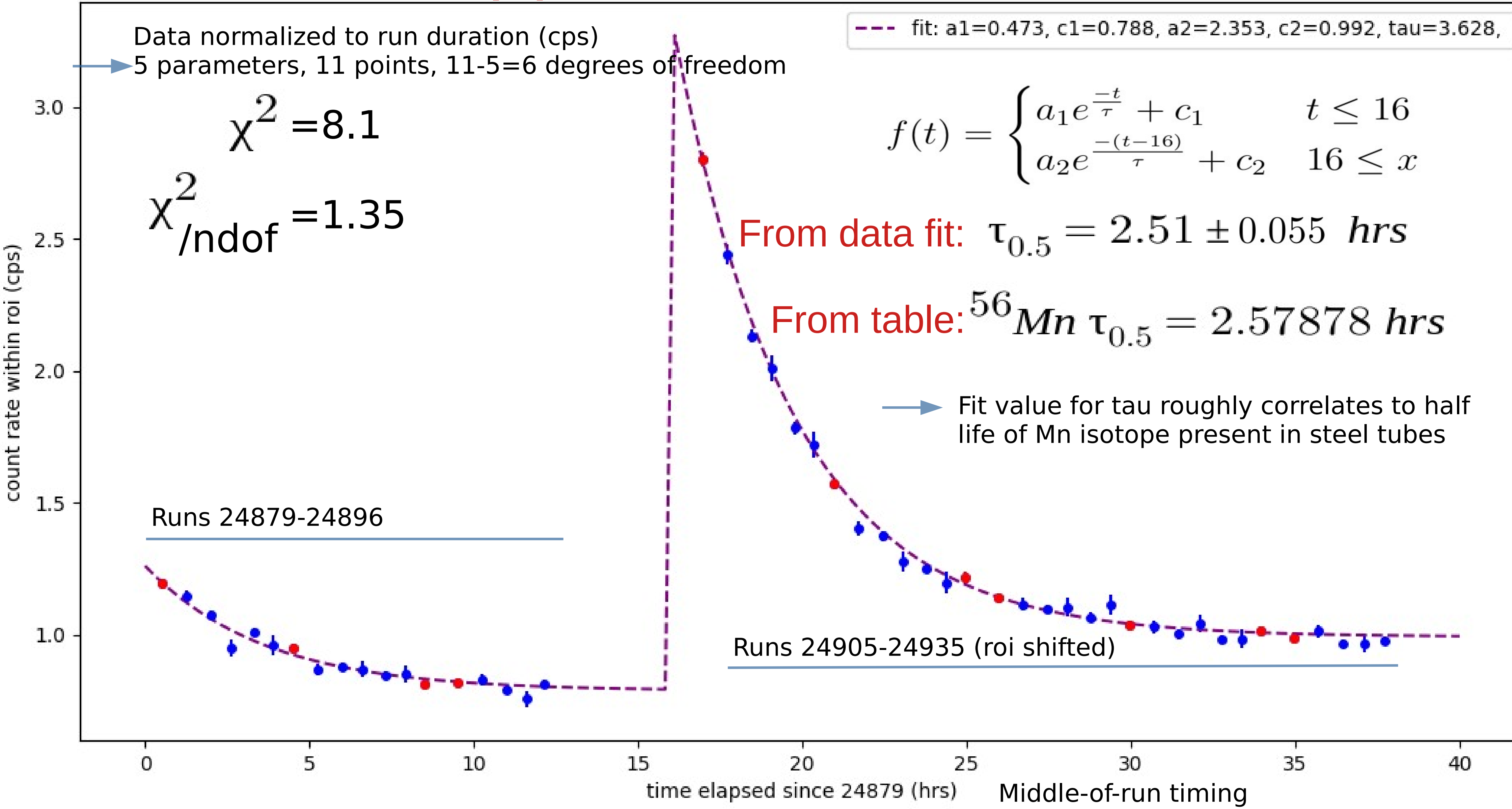




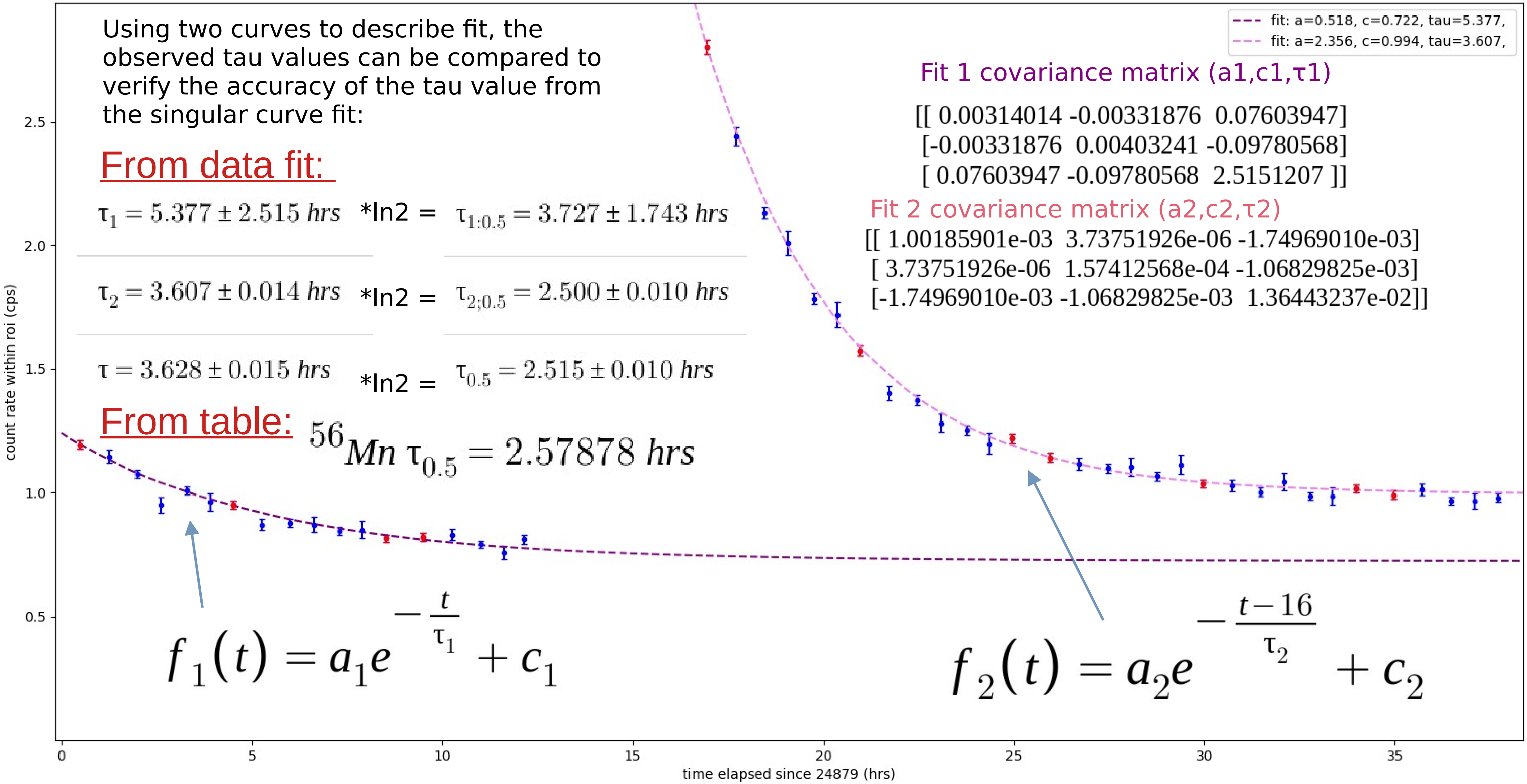
- Total counts per run within these roi were then recorded and plotted

Curve fitting to counts within ROI

Red runs have field |B|=0, curve is fit to red points **(assumes |B|=0 has no effect)**



Resilience of fit



Using two curves to describe fit, the observed tau values can be compared to verify the accuracy of the tau value from the singular curve fit:

From data fit:

$\tau_1 = 5.377 \pm 2.515\ hrs$ *ln2 = $\tau_{1;0.5} = 3.727 \pm 1.743\ hrs$

$\tau_2 = 3.607 \pm 0.014\ hrs$ *ln2 = $\tau_{2;0.5} = 2.500 \pm 0.010\ hrs$

$\tau = 3.628 \pm 0.015\ hrs$ *ln2 = $\tau_{0.5} = 2.515 \pm 0.010\ hrs$

From table: ${}^{56}Mn\ \tau_{0.5} = 2.57878\ hrs$

Fit 1 covariance matrix (a1,c1,τ1)

[[0.00314014 -0.00331876 0.07603947]
 [-0.00331876 0.00403241 -0.09780568]
 [0.07603947 -0.09780568 2.5151207]]

Fit 2 covariance matrix (a2,c2,τ2)

[[1.00185901e-03 3.73751926e-06 -1.74969010e-03]
 [3.73751926e-06 1.57412568e-04 -1.06829825e-03]
 [-1.74969010e-03 -1.06829825e-03 1.36443237e-02]]

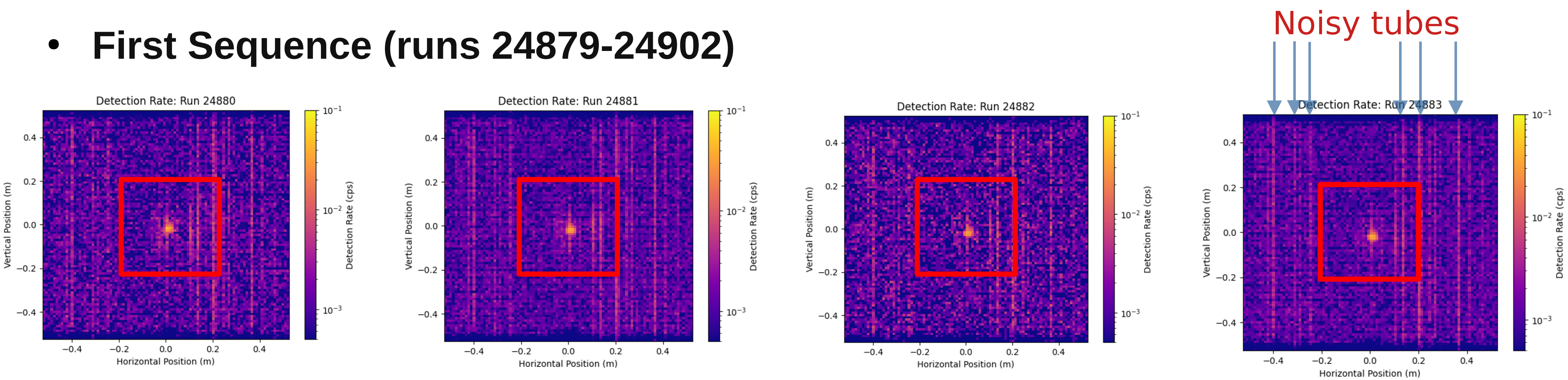
$$f_1(t) = a_1 e^{-\frac{t}{\tau_1}} + c_1$$

$$f_2(t) = a_2 e^{-\frac{t-16}{\tau_2}} + c_2$$

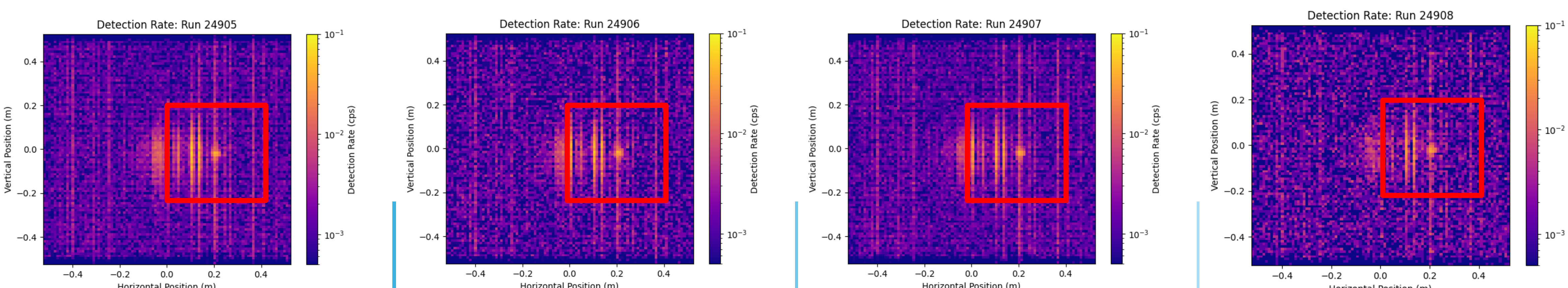
Regions of Interest

- Different ROIs include different tubes, some tubes are more noisy than others, could explain difference in background value (c1,c2) of fit

- **First Sequence (runs 24879-24902)**



Second Sequence (runs 24904-24935), **roi shifted 20cm**



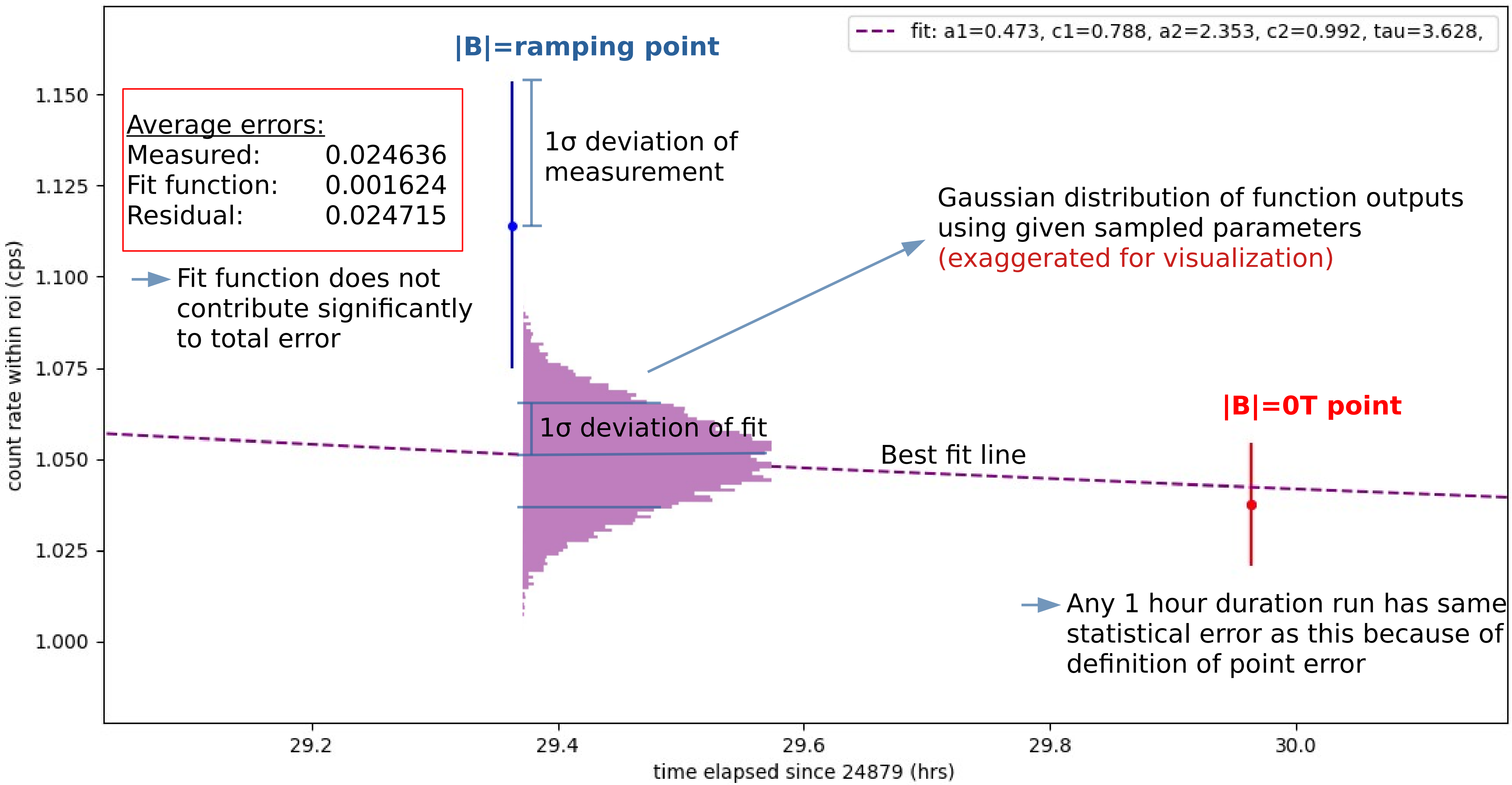
Covariance Matrix for |B|=0, 5 parameters fit

	a1	c1	a2	c2	tau
a1	9.74E-04	-3.31E-04	-3.24E-05	-2.02E-05	2.57E-04
c1	-3.31E-04	2.26E-04	6.71E-05	4.18E-05	-5.33E-04
a2	-3.24E-05	6.71E-05	1.12E-03	4.87E-07	-1.93E-03
c2	-2.02E-05	4.18E-05	4.87E-07	1.78E-04	-1.20E-03
tau	2.57E-04	-5.33E-04	-1.93E-03	-1.20E-03	1.53E-02

- Covariance matrix shows correlation between variables, diags show standard deviation of each variable
- symmetric positive definite matrix

$$f(t) = \begin{cases} a_1 e^{\frac{-t}{\tau}} + c_1 & t \leq 16 \\ a_2 e^{\frac{-(t-16)}{\tau}} + c_2 & 16 \leq x \end{cases}$$

Visualization of Sampling Process



Using cov. Matrix to find fit error

(1) For each time value, find estimated countrate using best fit function f(t) with optimal params

(2) take 5d correlated gaussian sample for params using cov, find f(t,p') estimated countrate given those params

(3) find difference between best fit estimate (1) and sampled params estimate (2)

(4) repeat steps (2)-(3) 100,000 times, find standard deviation of difference found in (3)

→ standard deviation = error for f(t) found in (1)

$$f(t) = \begin{cases} a_1 e^{\frac{-t}{\tau}} + c_1 & t \leq 16 \\ a_2 e^{\frac{-(t-16)}{\tau}} + c_2 & 16 \leq x \end{cases}$$

- Error for points = standard deviation of fit value for each point
- Using numpy function:

  np.random.mult(params, cov matrix)

  – Samples params for fit' function given covariance matrix from scipy fit

Error propagation for correction

- $measured - fitted = residual\ rate$

$R_m - R_f = R_r$ (Total measured – background effect) = isolated |B| effect we are looking for

- $\delta_m = \dfrac{\sqrt{N}}{\Delta t}$ Error for countrate value defined as sqrt(total counts within roi)/ total duration of run

  $\delta_f =$ Error for fit defined as 1 standard deviation of fit' value for each point, and fit'(t) calculated with parameters sampled from given covariance matrix

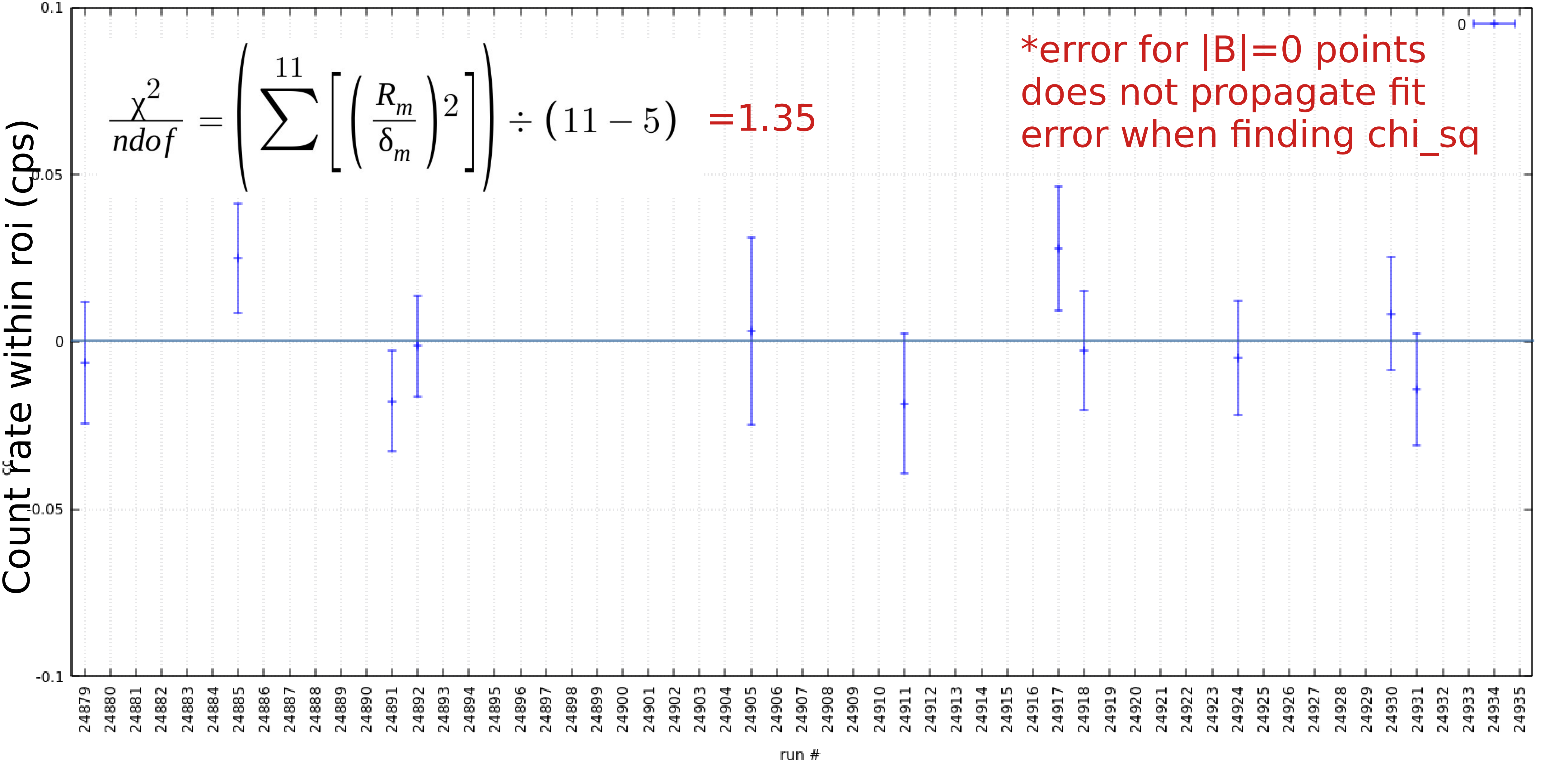
- $\delta_r = \sqrt{\delta_m^2 + \delta_f^2}$ Error for residual rate found by adding component rates in quadrature

- $R_r = (R_m - R_f) \pm \sqrt{\delta_m^2 + \delta_f^2}$ **Final value for residual rate**

Residue of |B|=0 points : finding chi^2

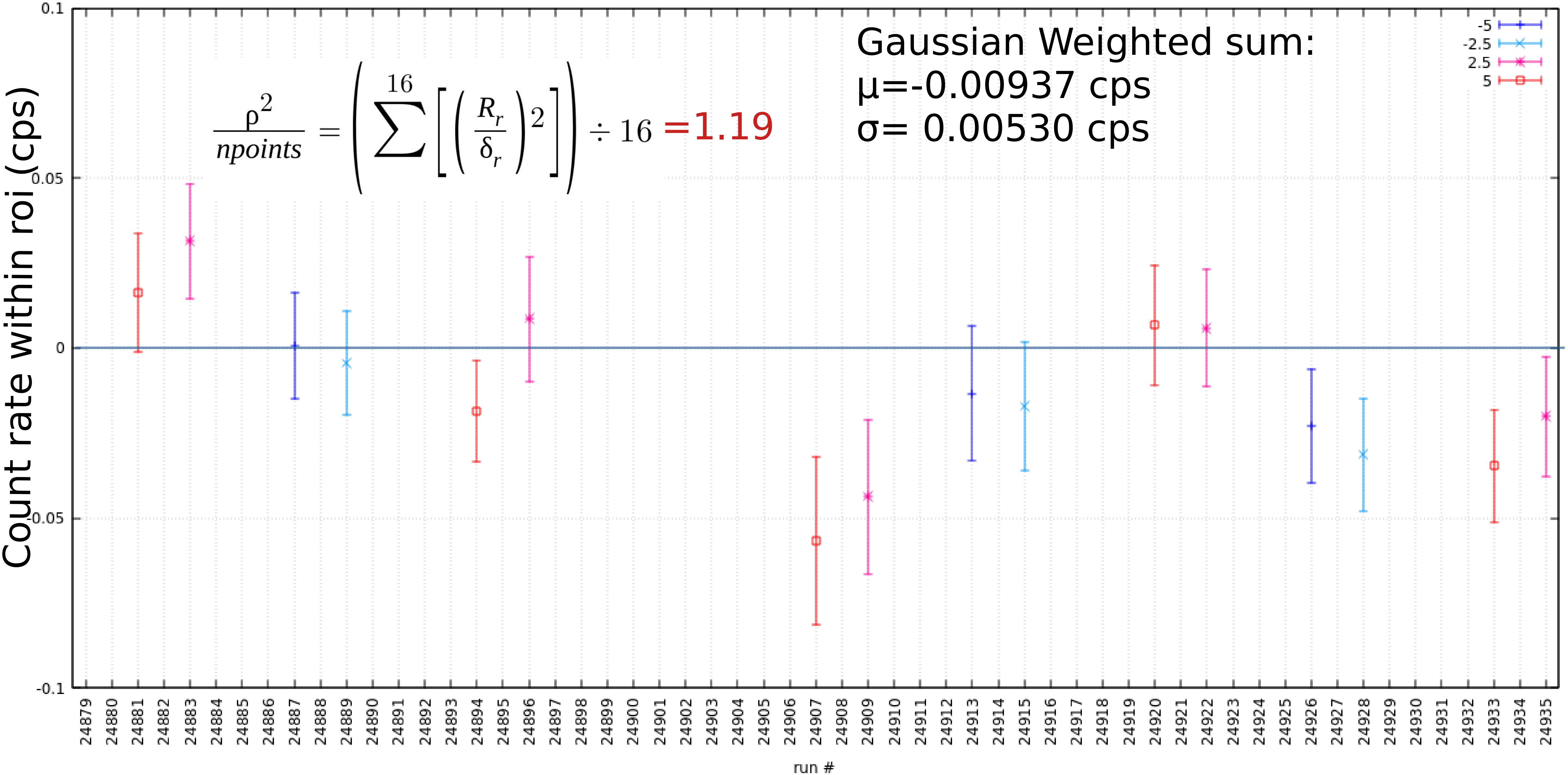
$$\frac{\chi^2}{ndof} = \left(\sum^{11} \left[\left(\frac{R_m}{\delta_m} \right)^2 \right] \right) \div (11-5)$$ =1.35

*error for |B|=0 points does not propagate fit error when finding chi_sq



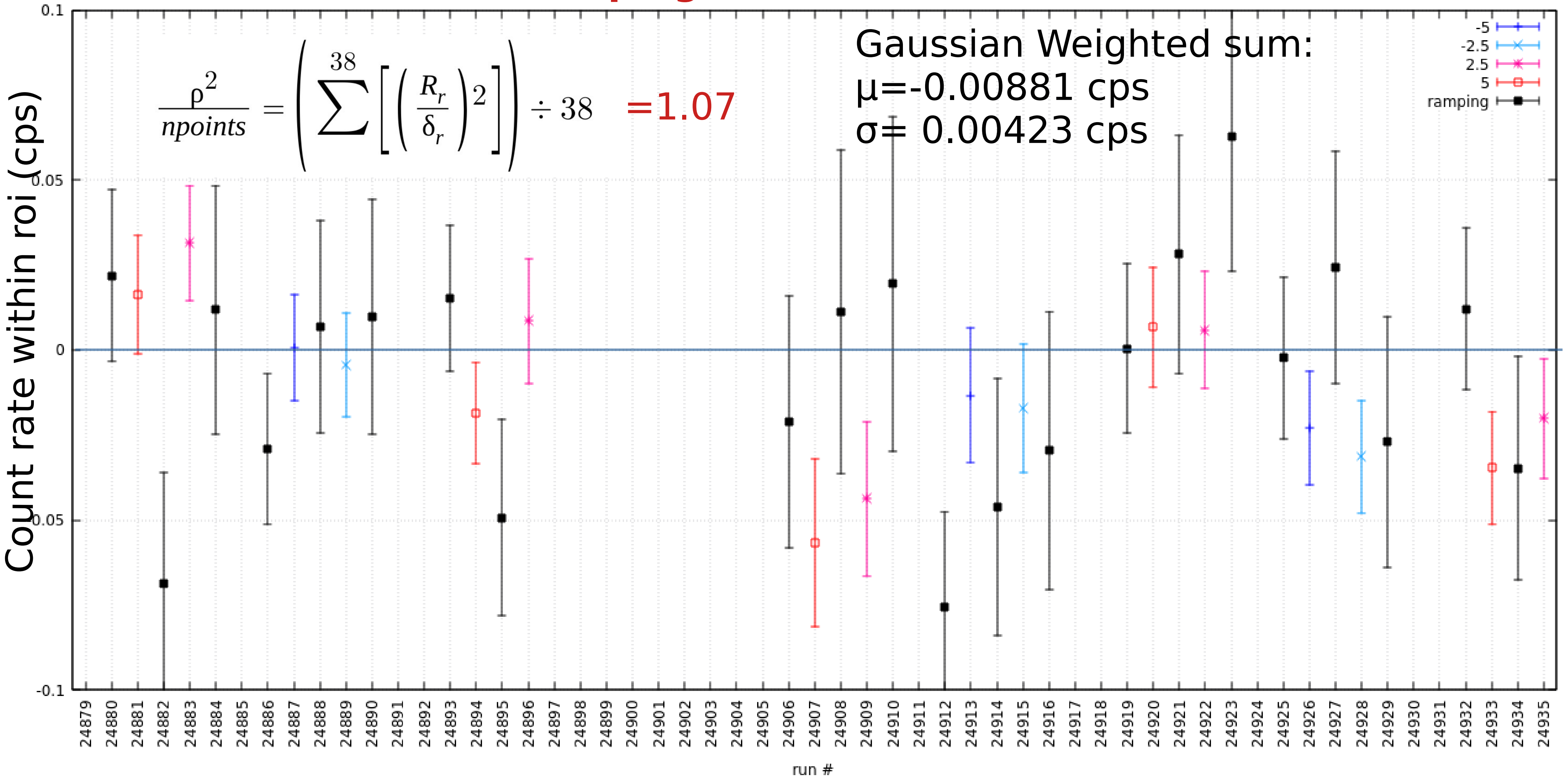
Residue with propagated error for runs |B|=-5,-2.5,2.5,5

$$\frac{\rho^2}{npoints} = \left(\sum^{16} \left[\left(\frac{R_r}{\delta_r} \right)^2 \right] \right) \div 16 = 1.19$$



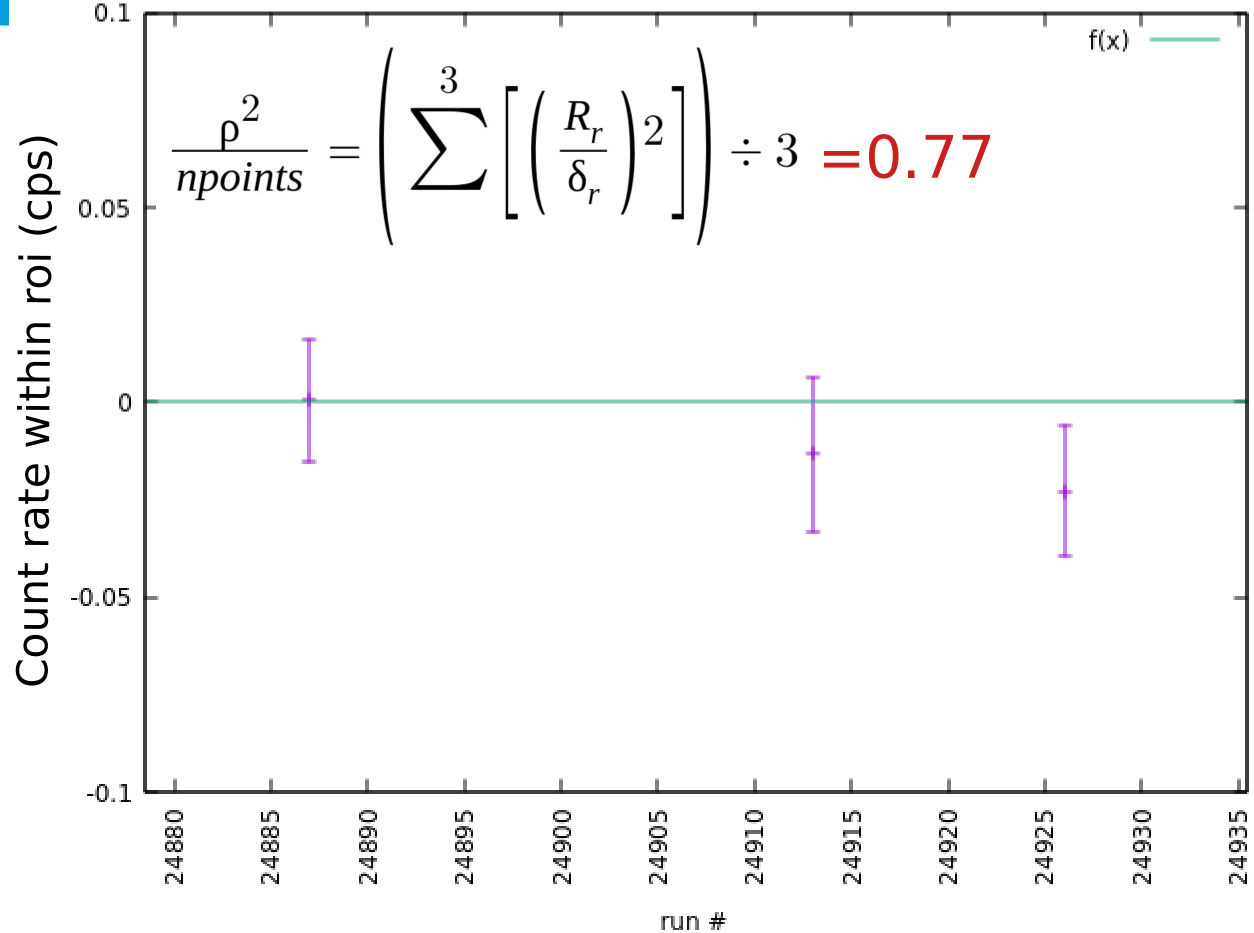
Residue with final error for runs |B|=/=0

***Ramping runs in black**



|B|=-5.0T

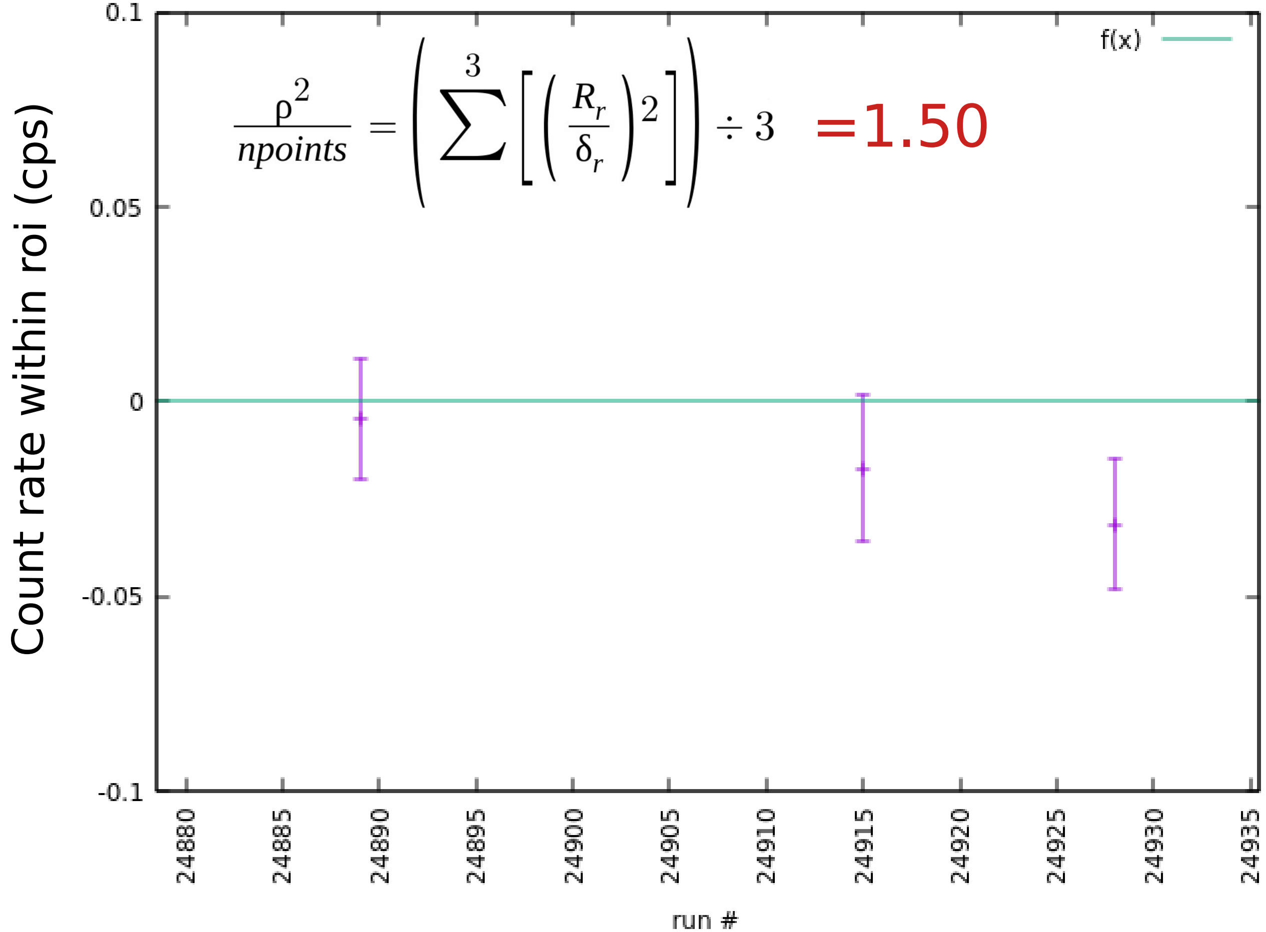
(residue per point)



Gaussian Weighted sum:
μ=-0.01147 cps
σ= 0.01188 cps

#run	corrected counts	error	|B|
24887	0.000595614738	0.0156598062	-5
24913	-0.0133341242	0.0198329428	-5
24926	-0.0227873336	0.0167062451	-5

|B|=-2.5T

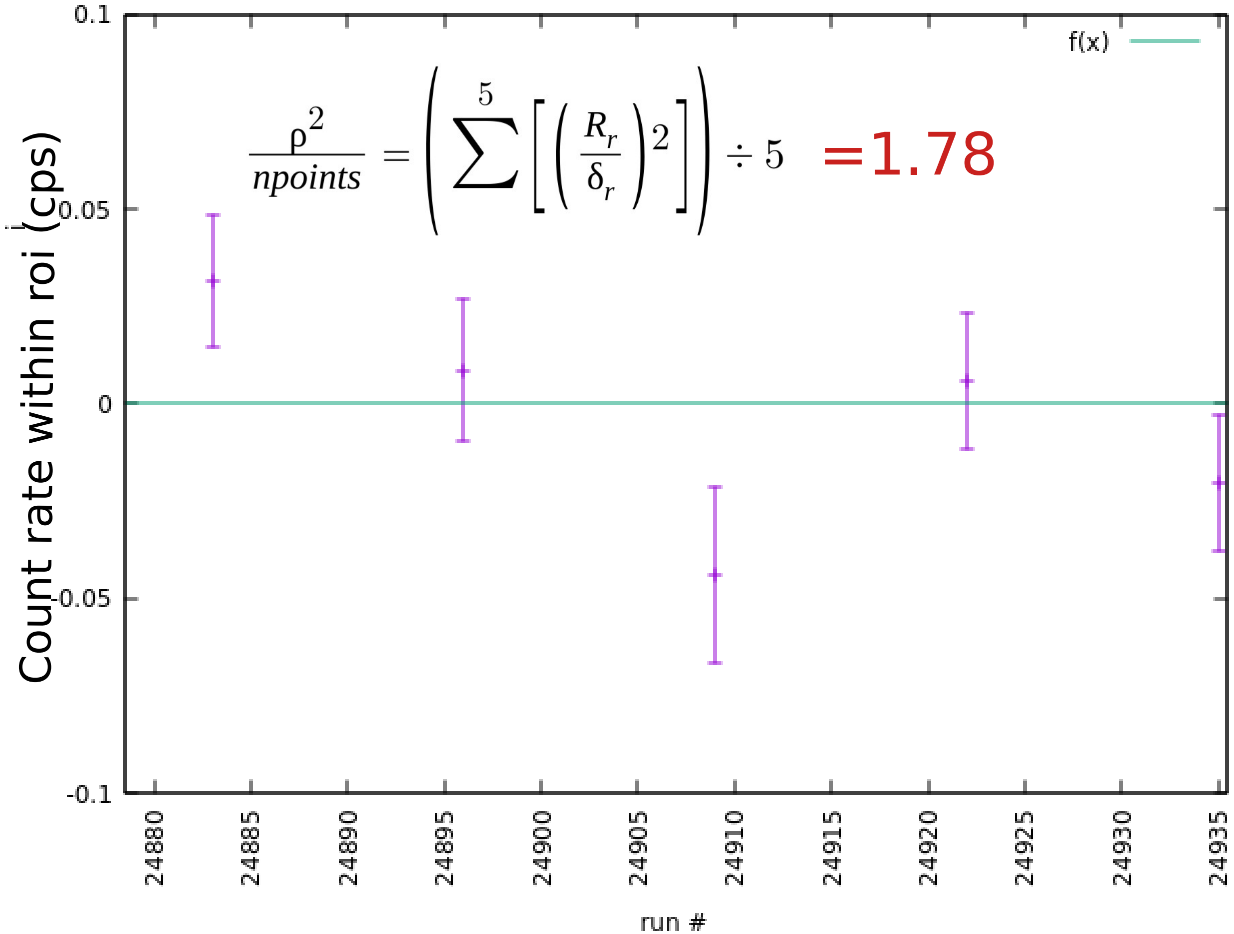


$$\frac{\rho^2}{npoints} = \left(\sum^{3} \left[\left(\frac{R_r}{\delta_r} \right)^2 \right] \right) \div 3 \quad = 1.50$$

Gaussian Weighted sum:
μ=-0.01731 cps
σ= 0.01180 cps

#run	corrected counts	error	|B|
24889	-0.00433777537	0.0153618211	-2.5
24915	-0.017016683	0.0188619339	-2.5
24928	-0.0313999617	0.0165443089	-2.5

|B|=2.5T

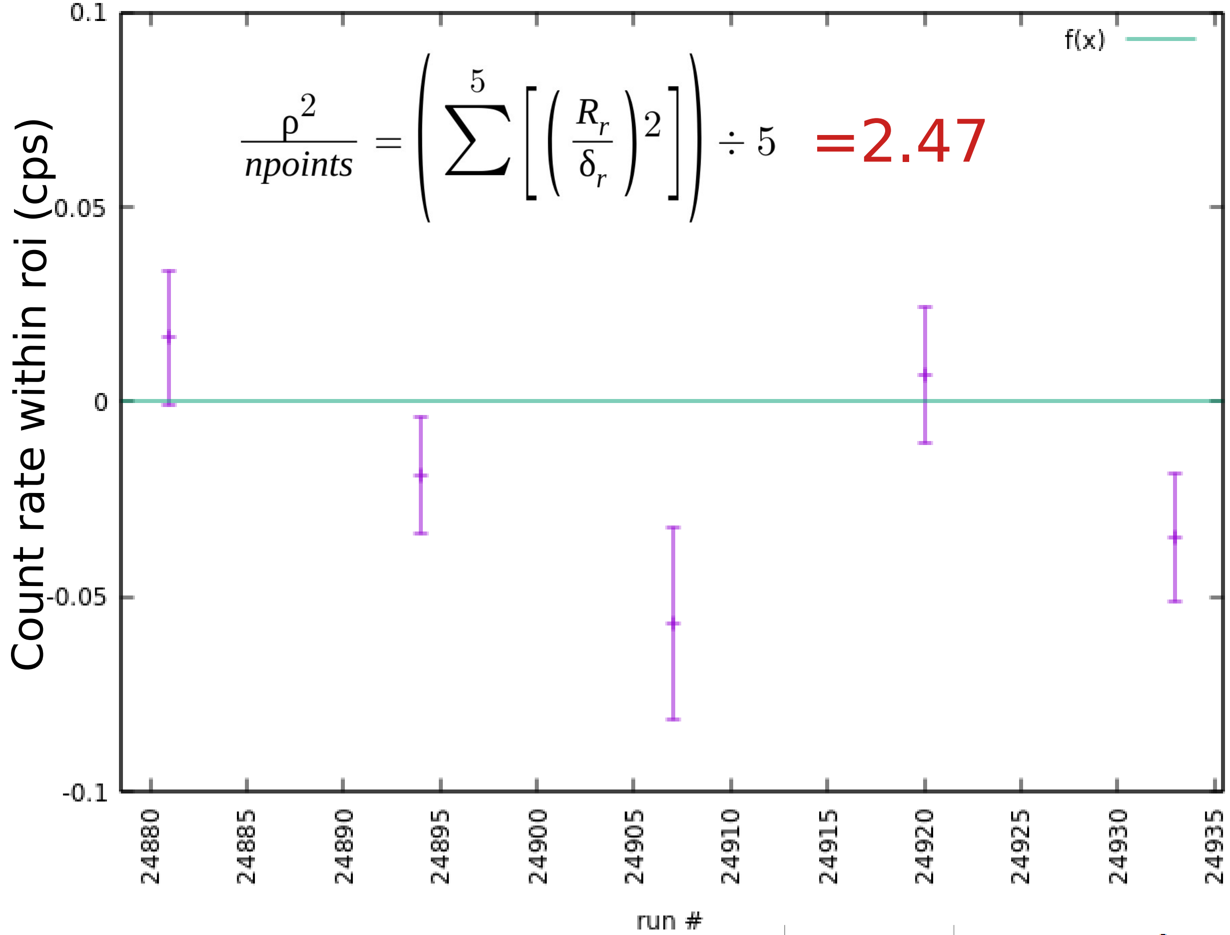


$$\frac{\rho^2}{npoints} = \left(\sum^{5} \left[\left(\frac{R_r}{\delta_r} \right)^2 \right] \right) \div 5 \quad =1.78$$

Gaussian Weighted sum:
μ=0.00138 cps
σ=0.00977 cps

#run	corrected counts within roi	error	|B|
24883	0.0314349476	0.0168019511	2.5
24896	0.00863990393	0.0183027262	2.5
24909	-0.0437128771	0.0226577458	2.5
24922	0.00585317692	0.0172674944	2.5
24935	-0.0201436788	0.0175670429	2.5

|B|=5.0T



$$\frac{\rho^2}{npoints} = \left(\sum^{5}\left[\left(\frac{R_r}{\delta_r}\right)^2\right]\right) \div 5 \quad =2.47$$

Gaussian Weighted sum:
μ=-0.01312 cps
σ= 0.00958 cps

#run	corrected counts within roi	error	|B|
24881	0.016430161	0.0173544826	5
24894	-0.0186506426	0.01484266	5
24907	-0.0565596898	0.0246868891	5
24920	0.00682909841	0.0175408731	5
24933	-0.0346497445	0.0163839068	5

Finding Probability/ neutron

Beam Intensity Measurements:
-Lowell Beam: 9.93 * 10^8 neutrons/s
-Au Foil: 7.4 * 10^8 neutrons/s

-Average Intensity: 8.7 * 10^8 neutrons/s
-Estimated Error: 1.3 * 10^8 neutrons/s
-Relative Error: 15%

Considering |B|=5.0T
Gaussian Weighted sum of corrected data:
μ=-0.01312
σ= 0.00958

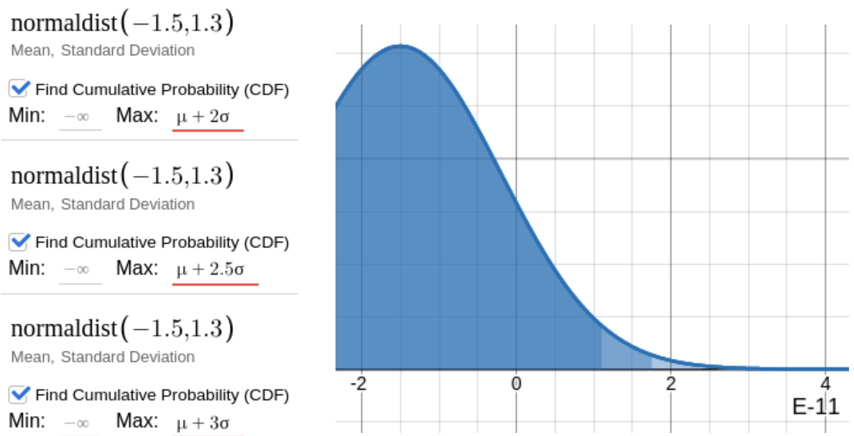
Gives:
Signal < 0.0026 [n/s] (95% Confidence Limit)
Signal < 0.0060 [n/s] (97.7% Confidence Limit)
Signal < 0.0092 [n/s] (99% Confidence Limit)

μ= - 0.01312 n/s
Intensity= 8.7* 10^8 n/s

μ/intensity = -1.5E-11 probability per neutron
σ/(intensity-estimated error) = 1.3E-11

Comparing to established limits

From previous slide:
μ/intensity = -1.5E-11 probability per neutron
σ/(intensity-estimated error) = 1.3E-11



Probability < 1.1 E-11 [1/n] (97.7% C.L.)
Probability < 1.75 E-11 [1/n] (99.4% C.L.)
Probability < 2.4 E-11 [1/n] (99.9% C.L.)

Found limit is ~10^-11/n, more accurate limit should be set with Feldman-Cousins approach

Thanks to Dr. Gonzalez for suggesting this format for C.L., also was recommended to use feldman-cousins statistics method or use poisson statistics to analyze data because of rarity of events

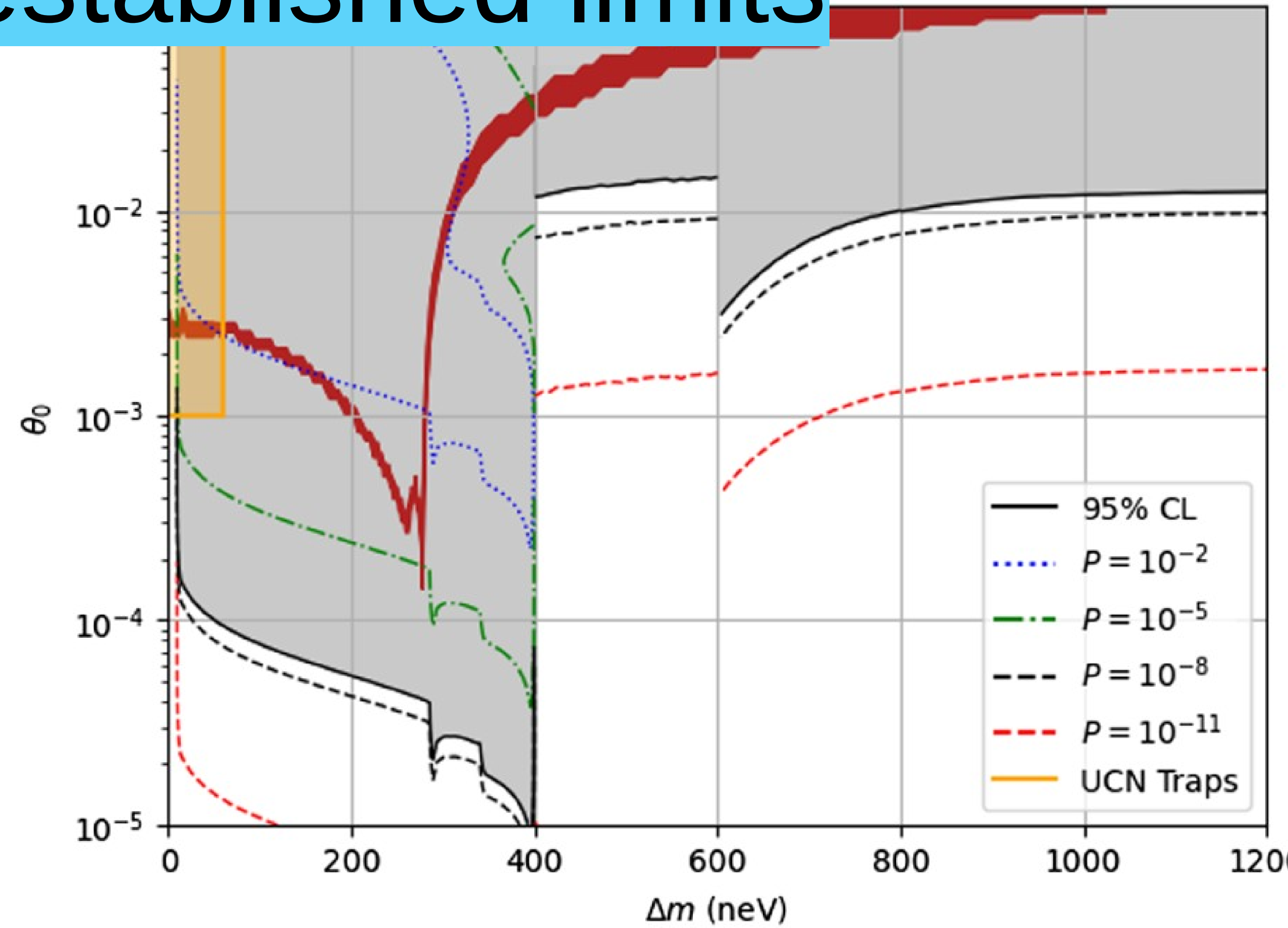


FIG. 3. The parameter space of the mirror matter model [19] excluded with 95% C.L. (gray region) and prediction consistent with the neutron lifetime anomaly (red band). Calculated probability of transmission is given by the dashed contour lines. The region 400–600 neV uses a different dataset.