B=0 analytic solutions for density matrix and for our PRL data

Analytic solution from paper
https://www.mdpi.com/2073-8994/14/2/230

symmetry MDPI

Article

Neutron-Mirror Neutron Oscillations in Absorbing Matter

Yuri Kamyshkov [1,*], James Ternullo [1], Louis Varriano [2] and Zurab Berezhiani [3,4]

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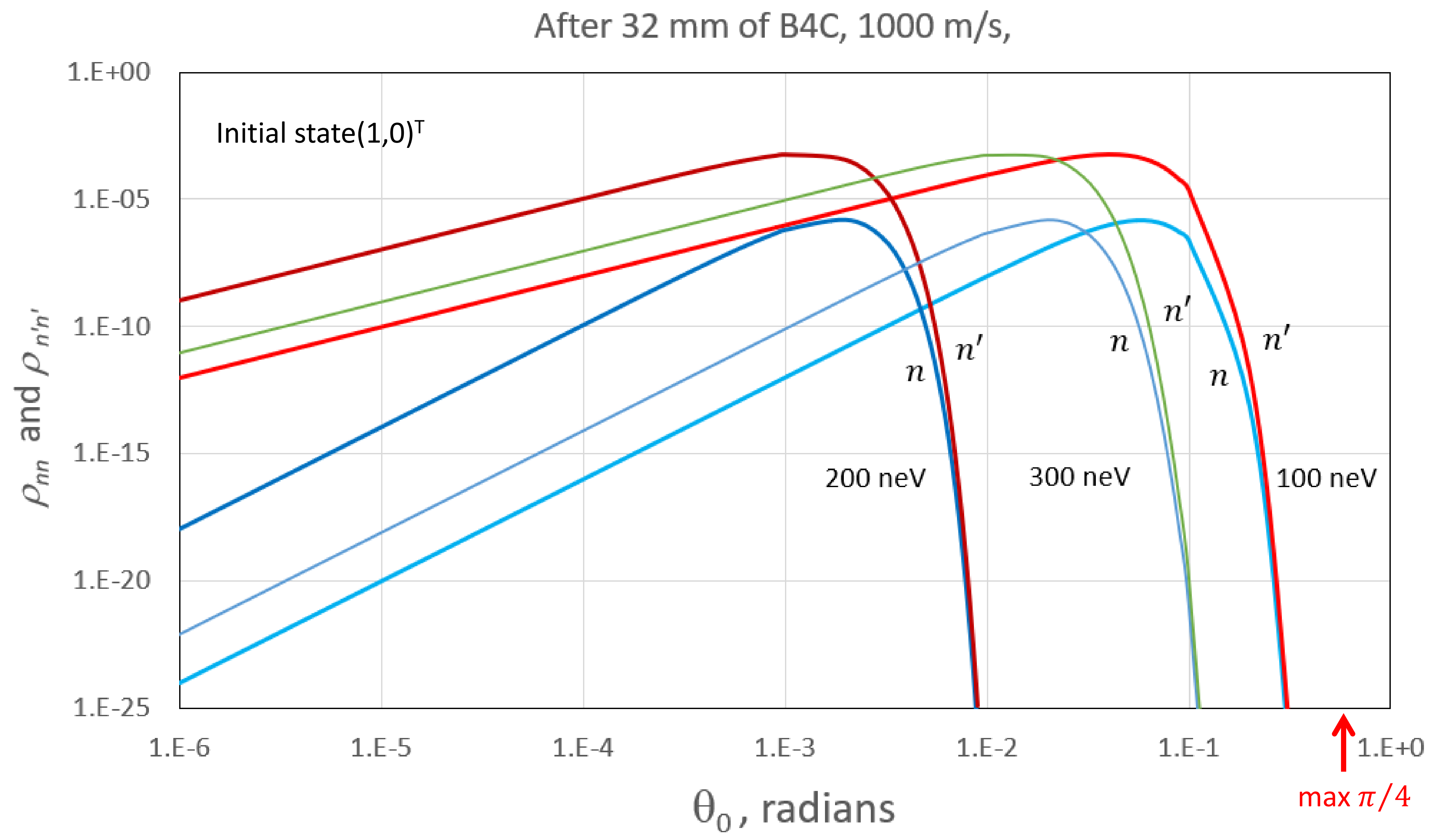
1. Extending to full region of possible mixing angle θ_0.

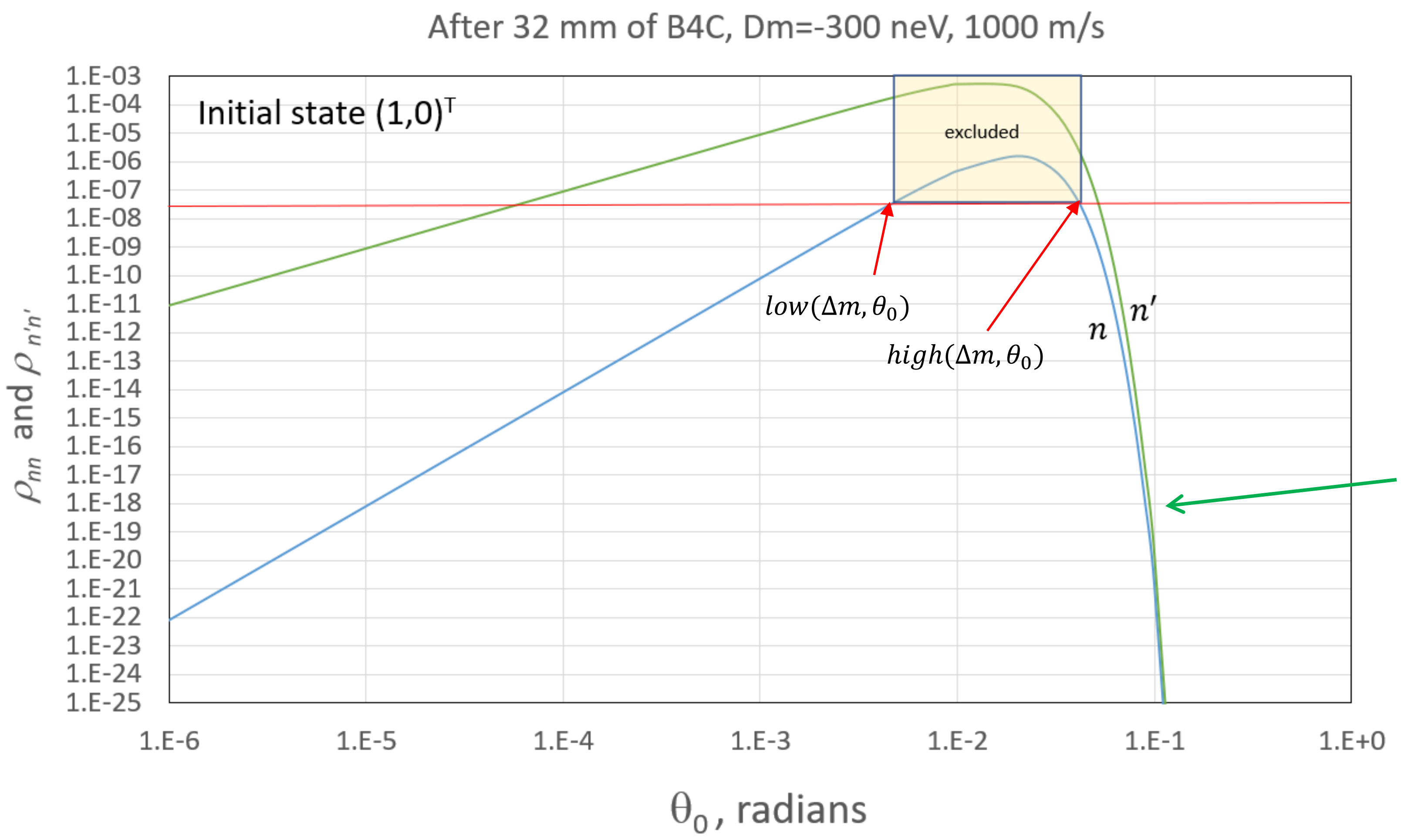
For small angles θ_0 in vacuum $P_{nn'} \cong 2\theta_0^2$.

$$\tan 2\theta_2 = \frac{2\epsilon}{U}$$

$U = \pm V_F \pm \Delta m \pm \mu B = 0$ - vacuum

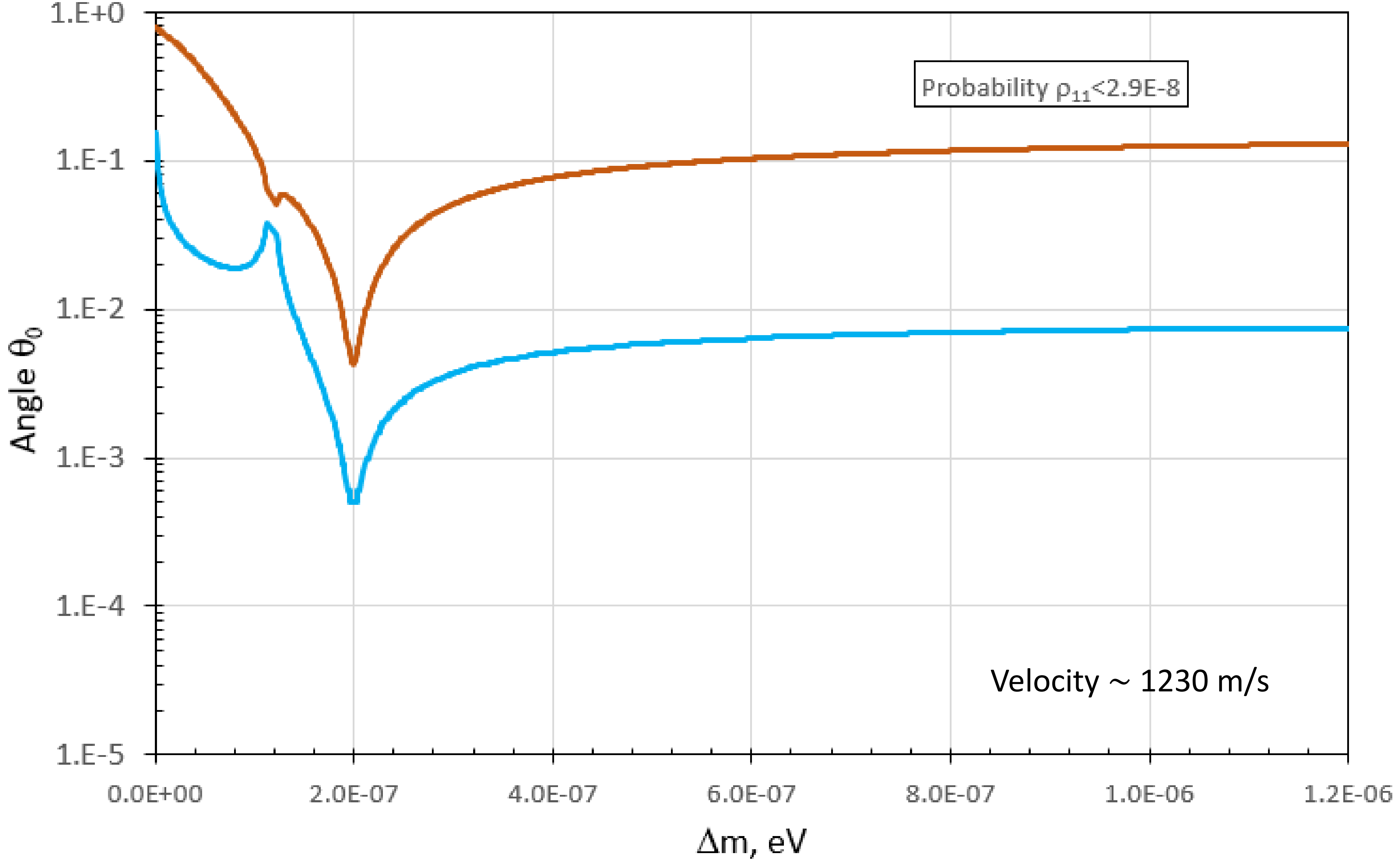
Range of possible mixing angles $0 < \theta_0 < 45°$


After 32 mm of B4C, 1000 m/s,
Initial state(1,0)T
1.E+00
1.E-05
1.E-10
1.E-15
1.E-20
1.E-25
ρnn and ρn'n'
n
n'
n
n'
n
n'
200 neV
300 neV
100 neV
1.E-6
1.E-5
1.E-4
1.E-3
1.E-2
1.E-1
1.E+0
θ0 , radians
max π/4



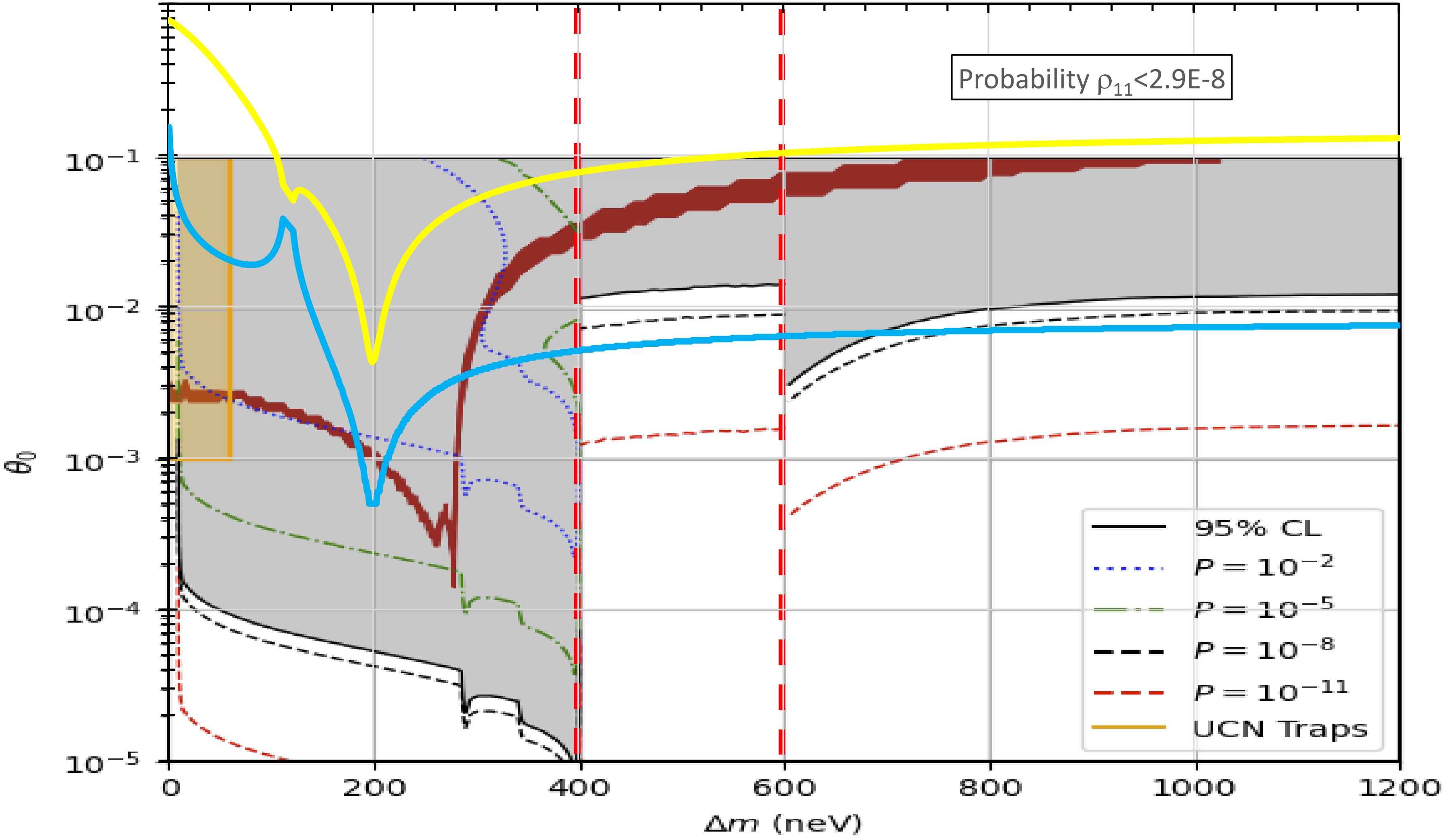
This fall-off is not a resonant effect but a general feature due to increasing of mutual oscillations

After 32 mm B_4C, B=0

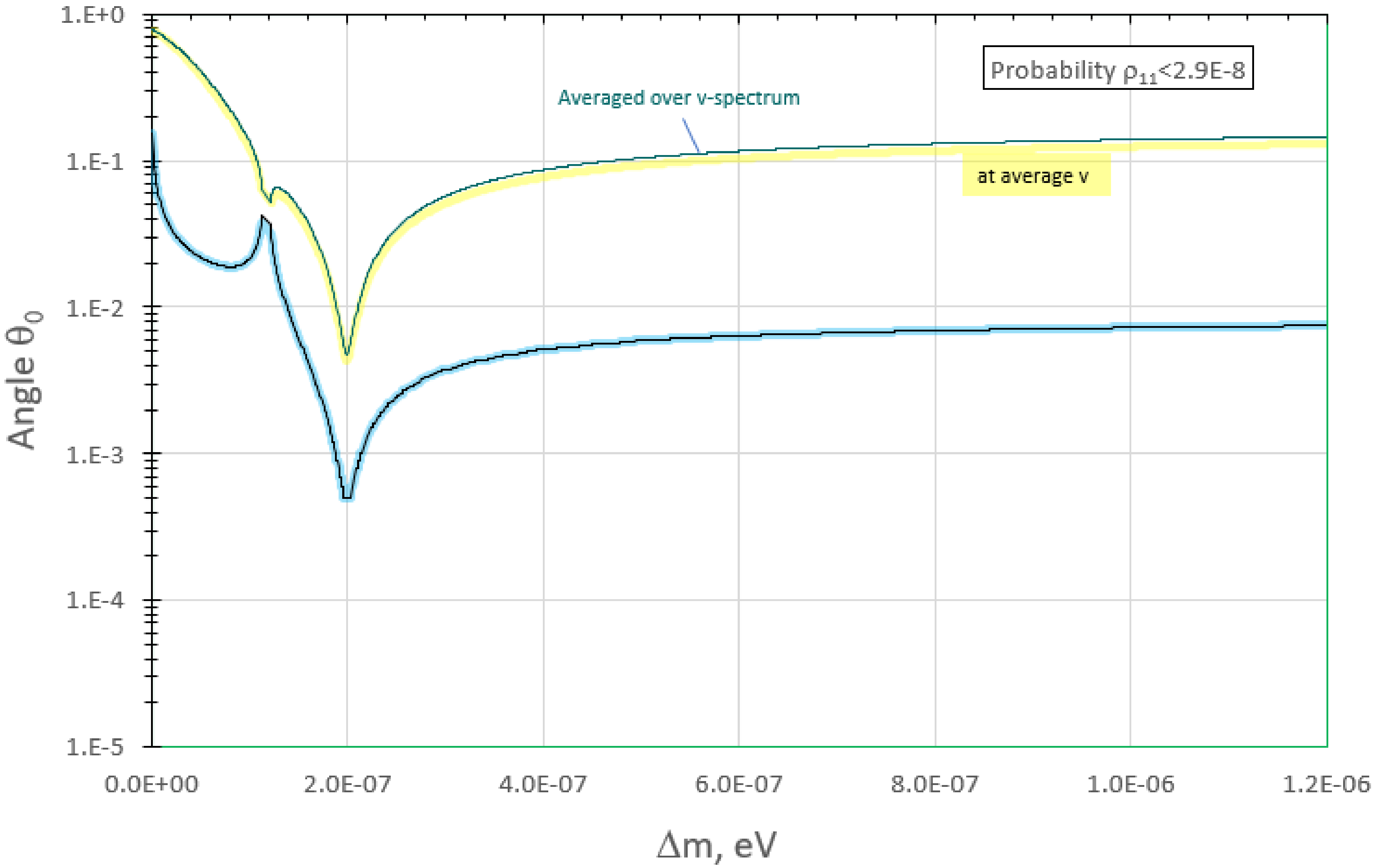


Implication for PRL paper in the region 400-600 neV where we used zero-field data B=0

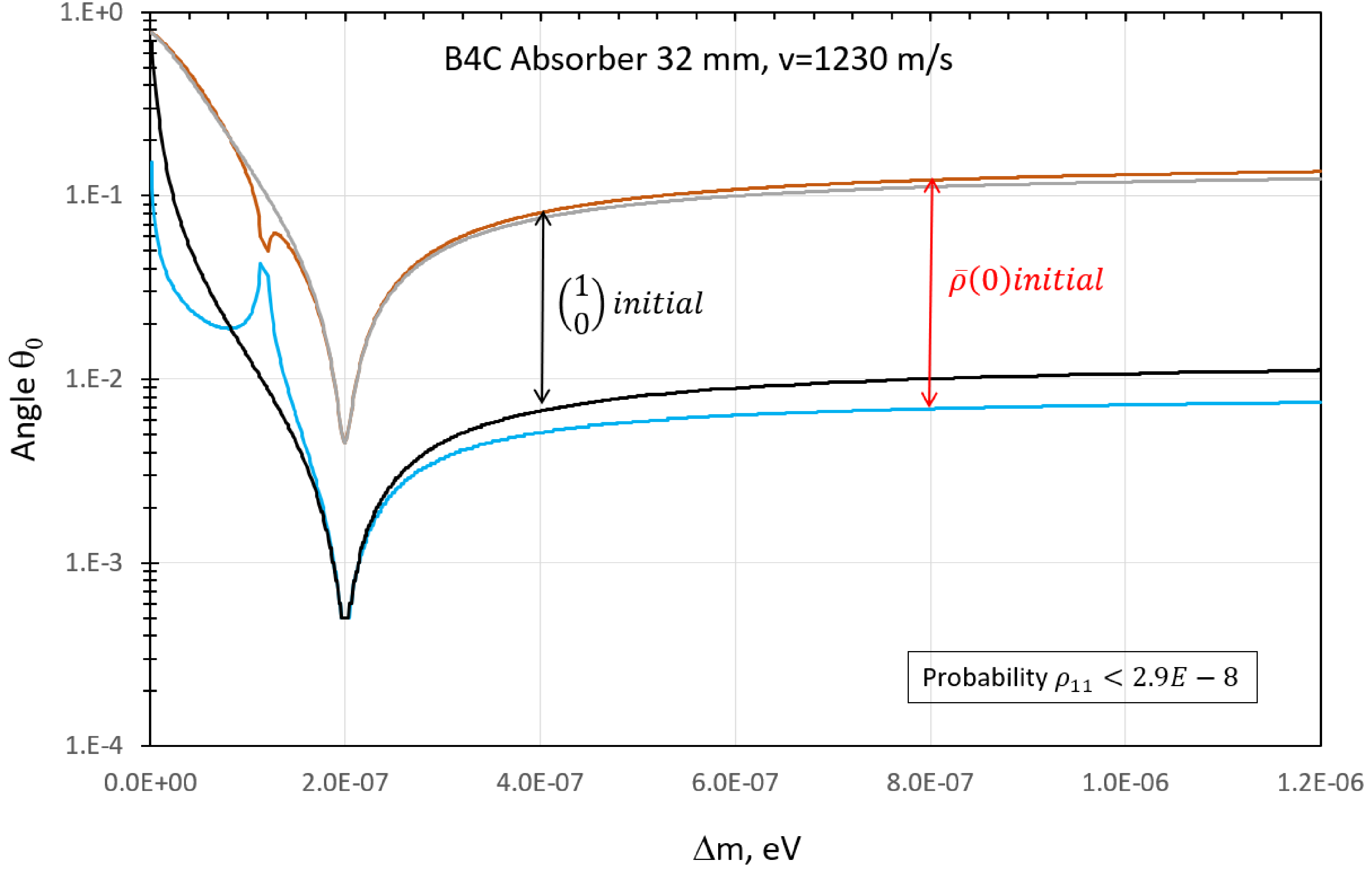
Velocity ~ 1230 m/s



Averaging over velocity spectrum gives ~ the same result as calculation at the average velocity (B=0; 1230 m/s for "3.75 Å", $\Delta m < 0$)

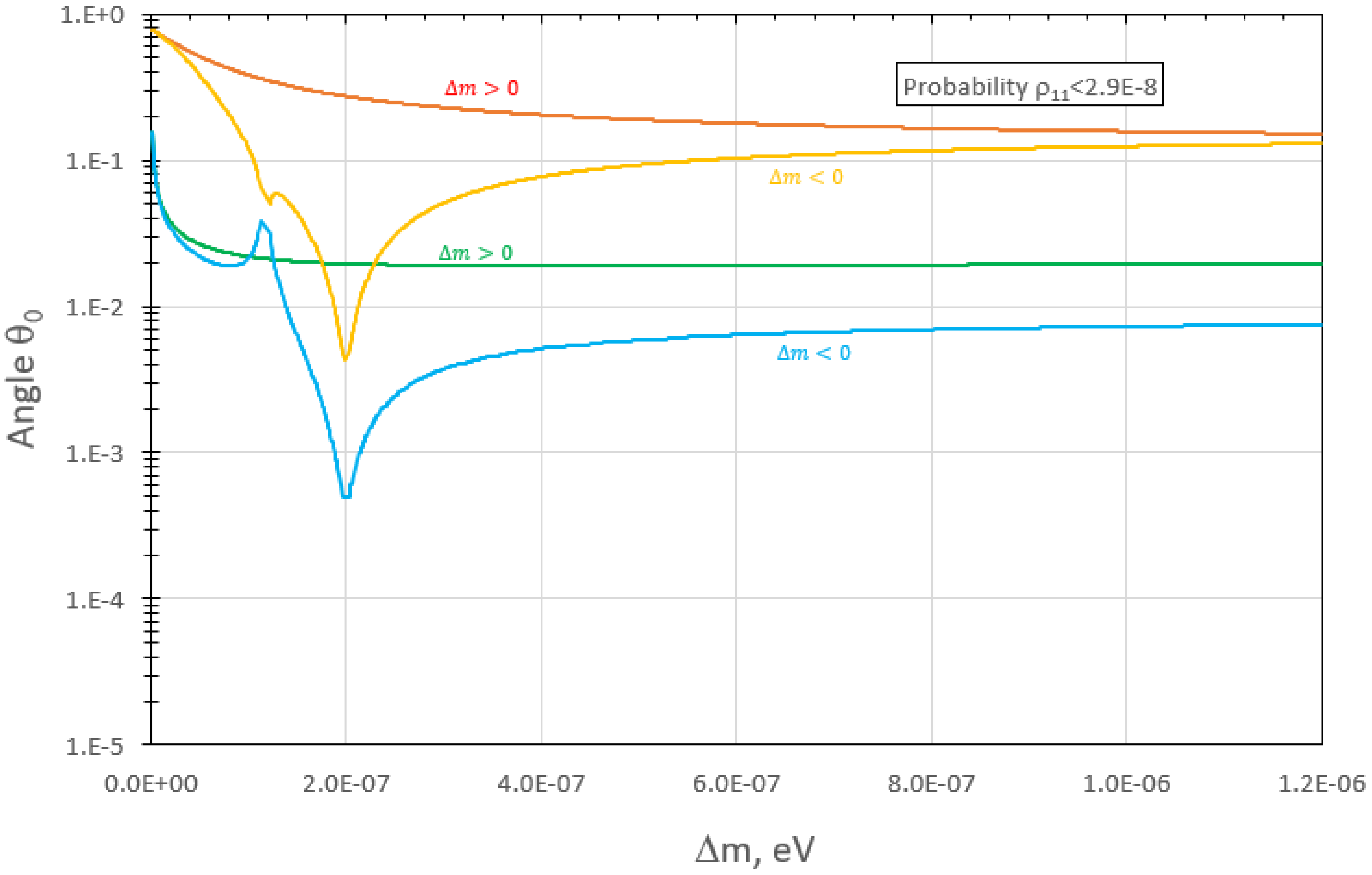


Strange effect of initial conditions
Velocity ~ 1230 m/s

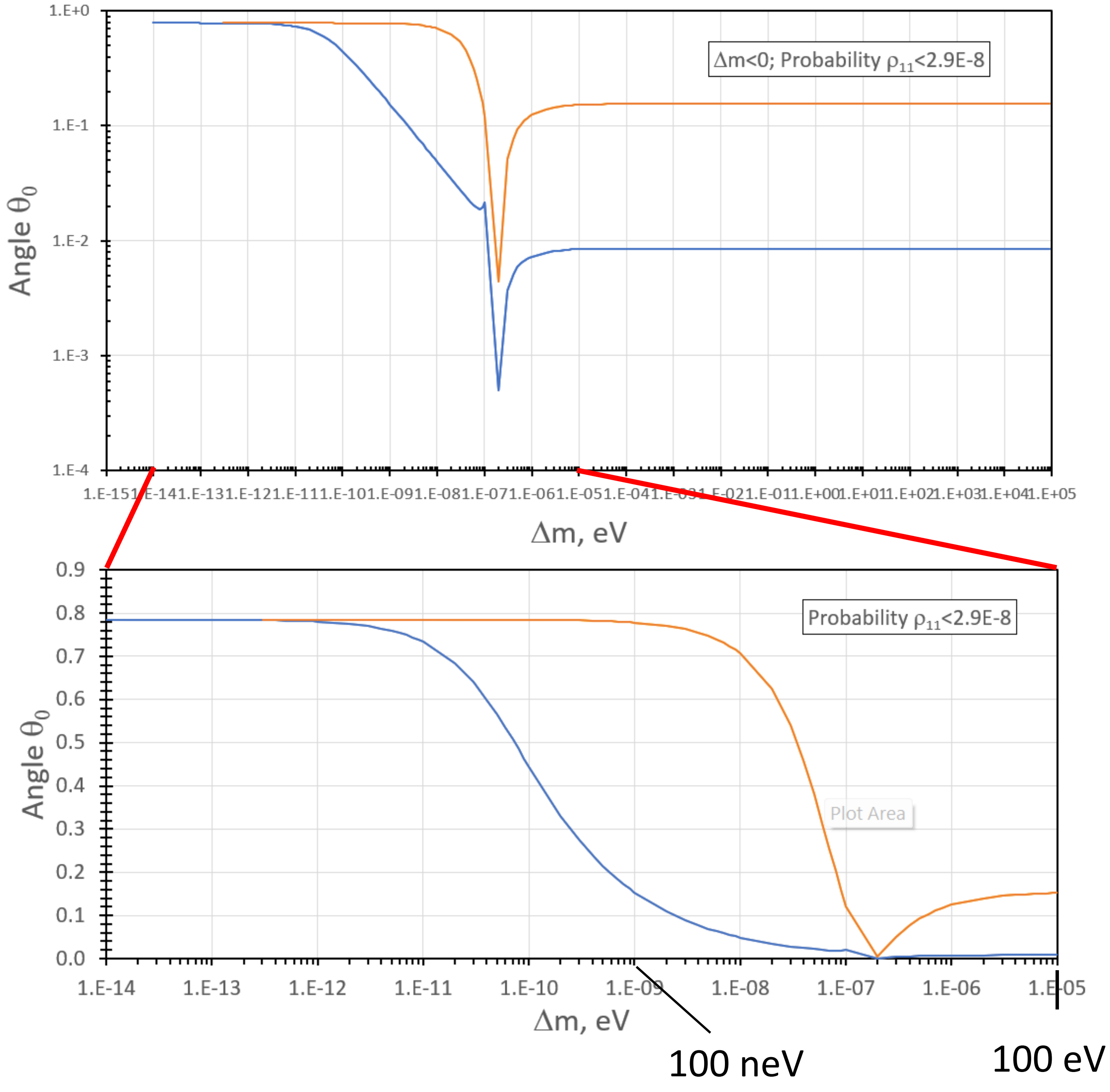


Negative (resonant) Δm and positive $\Delta m = m_n - m_{n'}$

Velocity ~ 1230 m/s



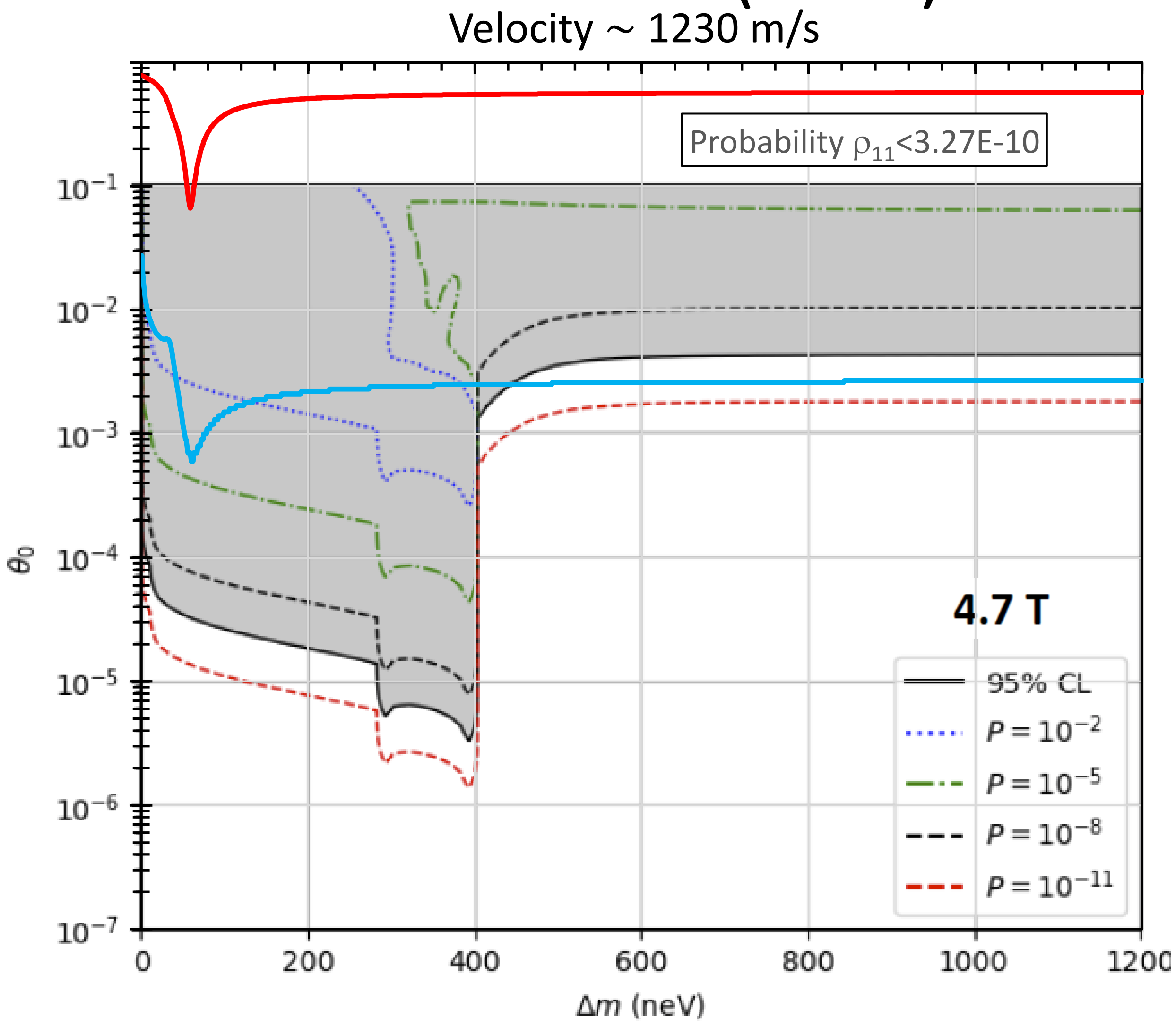
B4C 32 mm in a broader range of $\Delta m, B = 0$



Calculation for Cd (2020) Run

From Frank (Preliminary) →
For 2020 data

"Limits (Comparing to B=0 T)" ?



$$P(B = 4.7\ \mathrm{T}) = 3.27 \times 10^{-10}\ (95\%\ \mathrm{CL})$$

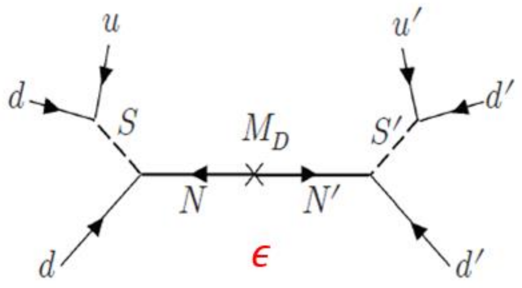
Conclusion

- Coverage of the $\Delta m - \theta_0$ exclusion plot for large angles $0.1 - 0.7854$ $(5° - 45°)$ with Magnetic Field ON should be separately studied. Everything for that is built in the density matrix evolution equations.

- How large can be Δm?

Operator of $n \to n'$ has dimension 9. That implies that $\epsilon \sim \frac{\Lambda_{QCD}^6}{\mathcal{M}^5} \sim \left(\frac{1TeV}{\mathcal{M}}\right)^5 \times 10^{-10}\ eV$

What the mass $\mathcal{M}$ can be ?



PDG limit on the mass of scalar di-quarks is $\mathcal{M} > 600\ GeV$. That corresponds to $\epsilon \sim 6 \times 10^{-10}\ eV$

Since $\theta_0 \sim \frac{\epsilon}{\Delta m}$ and $\theta \lesssim 0.01$ from our PRL paper for large masses: $\Delta m = \frac{6 \cdot 10^{-10}}{0.01} = 6 \cdot 10^{-8} eV = 60\ neV$

If we take the STEREO large Δm limit $\theta_0 < 4 \times 10^{-6}$ for the face value, then

$\Delta m = \frac{6 \cdot 10^{-10}}{0.000004} = 1.5 \cdot 10^{-4} eV = 150{,}000\ neV$. STEREO uses different model for possible large Δm

2. On Re/Im parts of $\rho_{12} = \rho_{21}^{*}$ in density matrix.

- Advantage of analytic solution – it is fast in calculations with complex numbers, but at interface of two media requires satisfy continuity of wave function and its derivative.

- Density matrix calculations are slow, but automatically continuous at the interface of two media with ρ_{12} containing the information about phases of motion.

a. Phase motion Im vs Re for unsuppressed oscillations in vacuum

$\theta_0 = 45°$ - maximum value $\quad H = \begin{pmatrix} 0 & \epsilon \\ \epsilon & 0 \end{pmatrix} \quad \tan 2\theta_0 = \frac{2\epsilon}{U}$

$U = \pm V_F \pm \Delta m \pm \mu B = 0$ - vacuum

$\psi_1 = \psi_n = \frac{1}{2}\left(e^{-itE_1} + e^{-itE_2}\right); \psi_2 = \psi_{n'} = \frac{1}{2}\left(e^{-itE_1} - e^{-itE_2}\right)$ - Solution of S.E. $\quad \Delta E = (E_1 - E_2)/\hbar$

$Initial\ density\ matrix$: $\hat{\rho}(t=0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ pure neutron

$\rho_{11}(t) = \rho_{nn}(t) = cos^2\left(\frac{\Delta E \cdot t}{2}\right)$ - averaged over time = 1/2

$\rho_{22}(t) = \rho_{n'n'}(t) = sin^2\left(\frac{\Delta E \cdot t}{2}\right)$ - averaged over time = 1/2

$\rho_{12}(t) = \rho_{nn'}(t) = \frac{1}{2} i\ sin(\Delta E t)$ - averaged over time = 0

$\rho_{21}(t) = \rho_{n'n}(t) = -\frac{1}{2} i\ sin(\Delta E t)$ - averaged over time = 0

$\bar{\rho} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$

$\rho_{12}(t)$ is pure Imaginary (no Real part of ρ_{12} that would mean suppression of oscillation amplitude)

For mixing angles close to max in absorbing media one should expect strong effect of non-resonant suppression for both n and n'

$$\rho_{nn}(t) = 1 - \sin^2 2\theta_0 \cdot \sin^2(\omega t), \quad \rho_{n'n'}(t) = \sin^2 2\theta_0 \cdot \sin^2(\omega t)$$

$$\rho_{nn'}(t) = -\frac{1}{2}\sin 4\theta_0 \cdot \sin^2(\omega t) - \frac{i}{2}\sin 2\theta_0 \cdot \sin(2\omega t), \quad \rho_{n'n}(t) = \rho^*_{nn'}(t)$$

$$\bar{\rho} = \begin{pmatrix} 1 - 2\theta_0^2 & -\theta_0 \\ -\theta_0 & 2\theta_0^2 \end{pmatrix}$$

No absorption for small θ_0

$$\bar{\rho}_{nn} = \left(1 - \tfrac{1}{2} \cdot \sin^2 2\theta_0\right) \cdot e^{-\Delta z/L_{air}}$$

$$\mathrm{Re}\bar{\rho}_{nn'} = \mathrm{Re}\bar{\rho}_{n'n} = -\tfrac{1}{4} \cdot \sin 4\theta_0 \cdot e^{-\Delta z/L_{air}}$$

$$\mathrm{Im}\bar{\rho}_{nn'} = \mathrm{Im}\bar{\rho}_{n'n} = 0$$

$$\bar{\rho}_{n'n'} = \tfrac{1}{2} \cdot \sin^2 2\theta_0 \cdot e^{-\Delta z/2L_{air}}$$

In air with absorption

Average density matrix is Real!

- Property of the average density matrix - it is repeating itself (?)
  Evolution $[\bar{\rho}] = \bar{\rho}$ (would make sense for non-absorptive H)

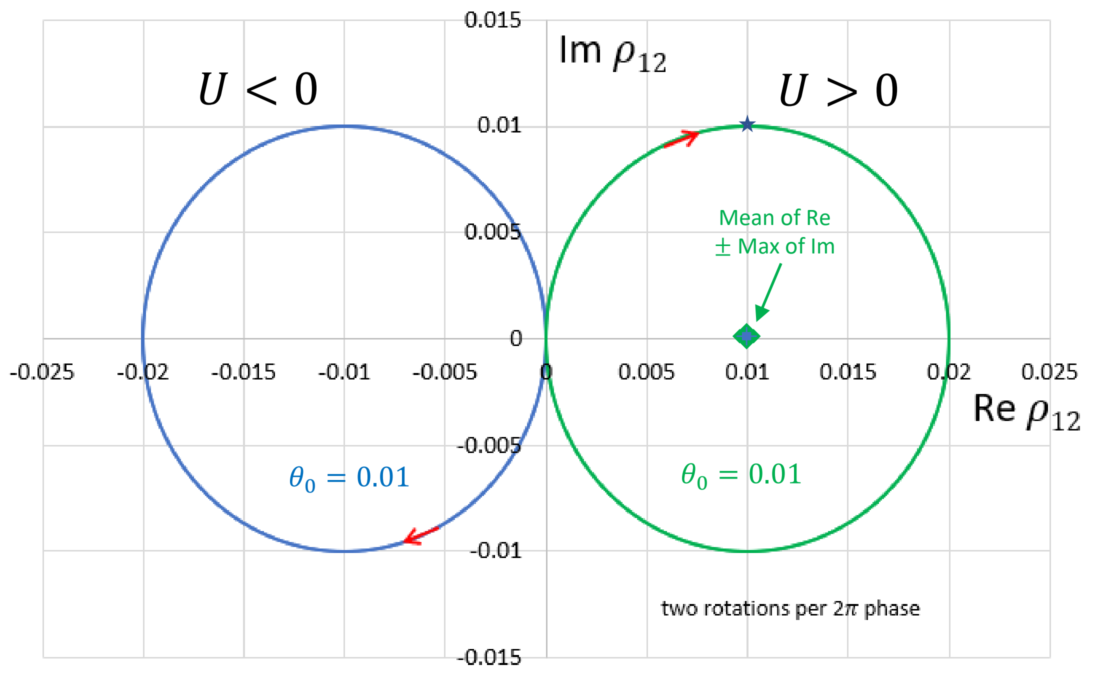
b. Phase motion Im vs Re for suppressed oscillations in non-absorptive media

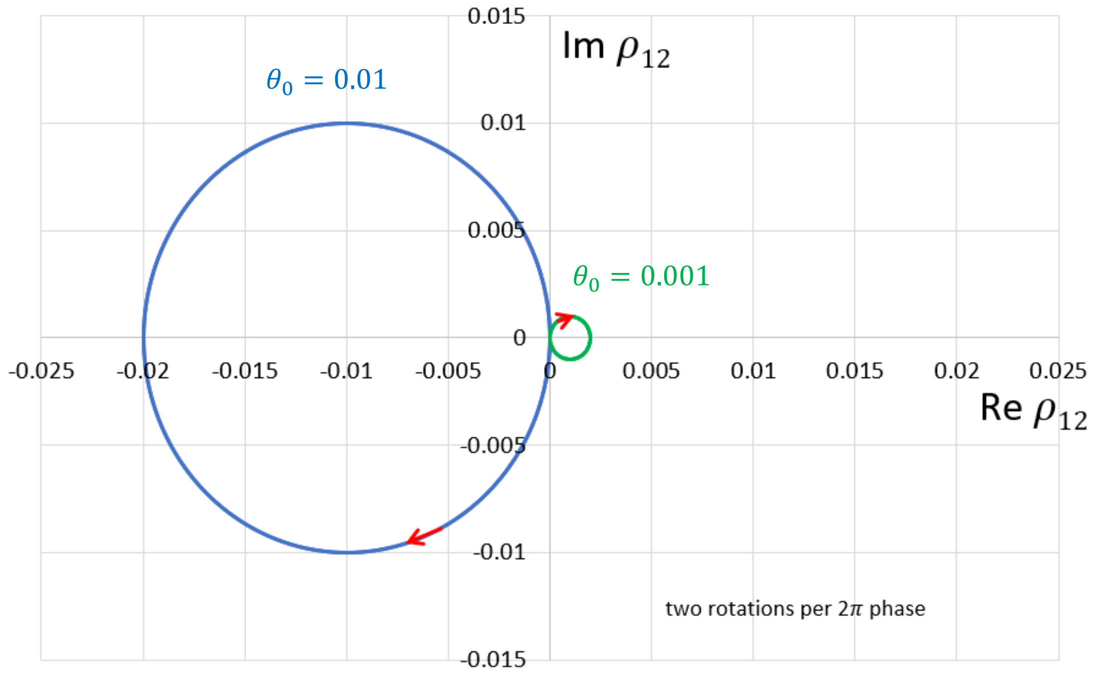
$$\tan 2\theta_0 = \frac{2\epsilon}{U} \qquad U = \pm V_F \pm \Delta m \pm \mu\Delta B \text{ - real suppression by } U$$

$$\rho_{nn}(t) = 1 - \sin^2 2\theta_0 \cdot \sin^2(\omega t), \quad \rho_{n'n'}(t) = \sin^2 2\theta_0 \cdot \sin^2(\omega t)$$

$$\rho_{nn'}(t) = -\frac{1}{2}\sin 4\theta_0 \cdot \sin^2(\omega t) - \frac{i}{2}\sin 2\theta_0 \cdot \sin(2\omega t), \quad \rho_{n'n}(t) = \rho^*_{nn'}(t)$$

$$sin^2 2\theta = \frac{tan^2 2\theta}{1 + tan^2 2\theta} = \frac{4\epsilon^2}{(U^2 + 4\epsilon^2)}$$





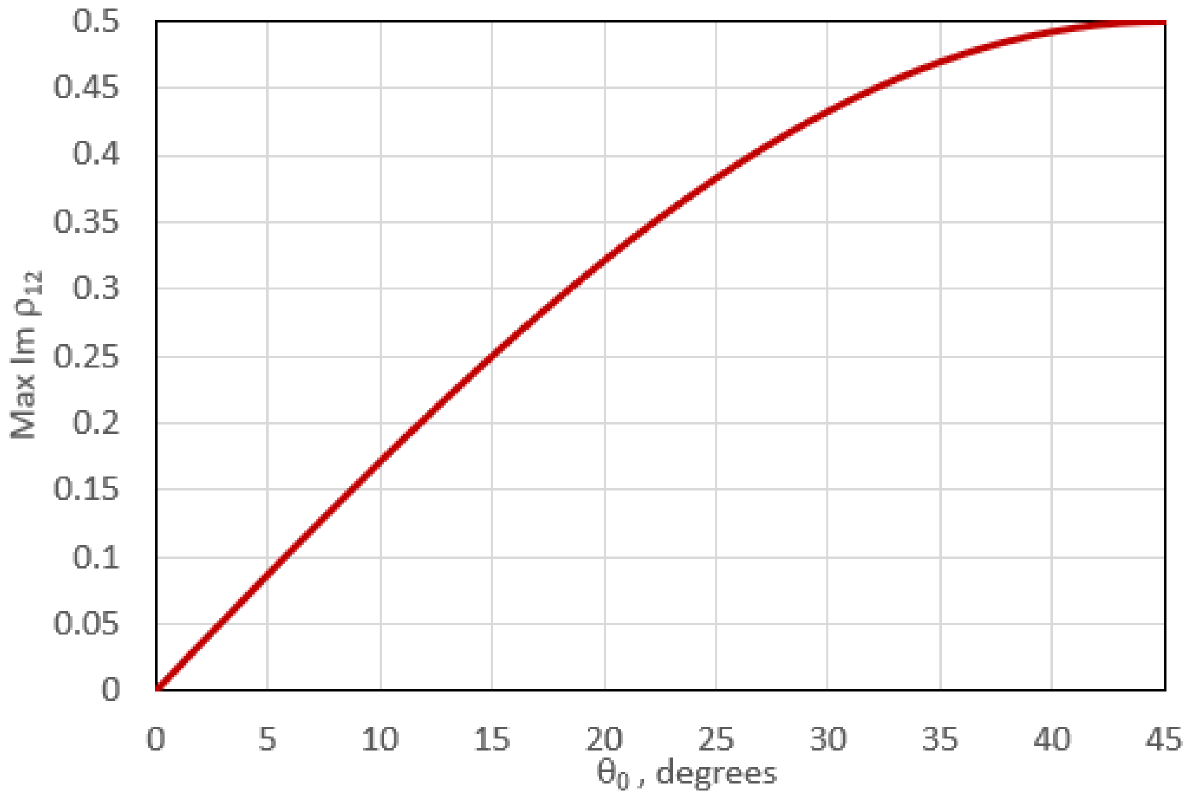
- evolution of the averaged density matrix is the average of the evolutions of two systems motion with $\pm t$ for the phase $\pi/4$ from $\rho_{12} = \rho_{21} = 0$ $(with\ Re = 0, Im = 0)$

Magnitude of $Im\ \rho_{12} = -Im\ \rho_{21}$ (no absorption)

$$Im\ \rho_{21} = \frac{1}{2}\sin 2\theta_0 \cdot \sin 2\omega t = \frac{\epsilon}{\sqrt{U^2 + 4\epsilon^2}} \cdot \sin 2\omega t < 0.5 \cdot \sin 2\omega t$$

$$\frac{1}{2}\sin 2\theta_0 = \frac{\epsilon}{\sqrt{U^2+4\epsilon^2}} \rightarrow$$

$Max\ Im\ \rho_{21} = 1/2$



$$\overline{Im\ \rho_{21}} = 0$$

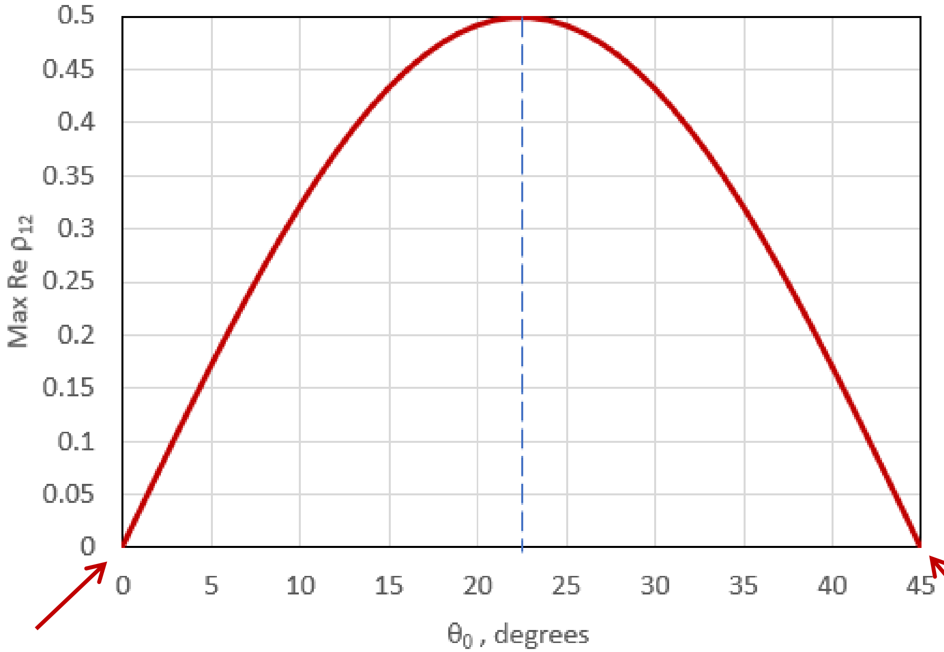
Means that measurements of averaged probability do not oscillate

Magnitude of $Re\ \rho_{12} = Re\ \rho_{21}$

$$sin^2 2\theta = \frac{4\epsilon^2}{(U^2 + 4\epsilon^2)}$$

$$Re\ \rho_{12} = \pm\frac{1}{2}\sin 4\theta_0 \cdot \sin^2(\omega t) = \frac{2\epsilon U}{U^2+4\epsilon^2} \cdot \sin^2(\omega t)$$

$Max\ Re\ \rho_{12}$=1/2

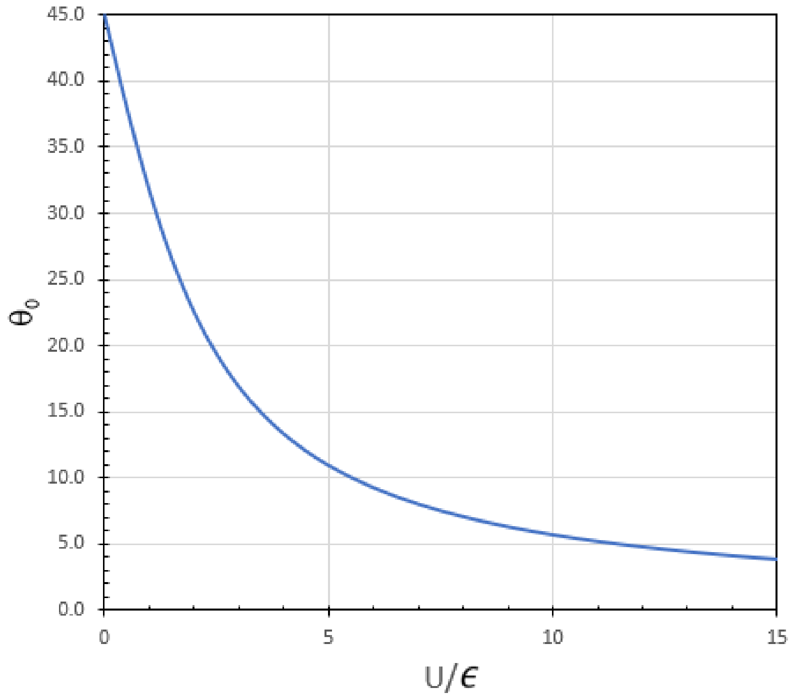


$Re\ \rho_{12} = 0$ for $U \to \infty$

$Re\ \rho_{12} = 0$ for $\theta_0 = 45°$
free unsuppressed oscillations

$$\overline{Re\ \rho_{12}} = \frac{\epsilon U}{U^2 + 4\epsilon^2}$$

$$Max\ \overline{Re\ \rho_{12}} = 1/4$$



$$22.5° = \frac{\theta_0(max)}{2} \to U = 2\epsilon$$

Conclusion for understanding of the average density matrix

- Time-dependent Schrodinger equation for two-level system gives time evolution for $\psi_1\,(t)$ and $\psi_2\,(t)$ wave functions with the relative phases that comprises the oscillation effect. Density matrix $\rho(t)$ can be calculated from that.

- Physical measurement means averaging including averaging over phases of system motion . That can be done with known ψ_1 and ψ_2 or with known $\rho(t)$. It requires the averaging process. It can be performed as phase averaging for the high frequency of oscillation $\left(\overline{sin^2\omega t} = 1/2\right)$ or at given t over more general averaging over the velocity spectrum.

- At the border of two media wave functions should be continuous together with its derivatives (boring procedure) and can be averaged only after that. Evolution of density matrix is continuous.

- Averaged density matrix doesn't represent wave functions of one of trajectories of the motion, but can be represented by two motions around the point t of averaged density matrix $\rho(t)$ for equal phases $\pm\,\pi/4$.

- Analytic calculations probabilities using solution of time-dependent Schrodinger equation for the interface of two media can not be accurately done with the averaged density matrix (will continue working on that).