April 12, 2022. NN’ group meeting. YK

How analytic solution works for

[Neutron-Mirror Neutron Oscillations in Absorbing Matter](https://www.mdpi.com/2073-8994/14/2/230)

from our paper

https://www.mdpi.com/2073-8994/14/2/230

$$\mathscr{H} = \begin{pmatrix} U - iW & \epsilon \\ \epsilon & 0 \end{pmatrix} = \begin{pmatrix} V - \Delta m \pm \mu B - iW & \epsilon \\ \epsilon & 0 \end{pmatrix}$$

$$\mathscr{H} = \mathscr{H}_{\rm osc} + \mathscr{H}_{\rm abs}\,, \quad \mathscr{H}_{\rm osc} = \begin{pmatrix} U & \epsilon \\ \epsilon & 0 \end{pmatrix} \quad \mathscr{H}_{\rm abs} = -i\begin{pmatrix} W & 0 \\ 0 & 0 \end{pmatrix}$$

Hamiltonian is a sum of hermitian and anti-hermitian parts

It can be diagonalized by "canonical" transformation: $S\,\mathscr{H}\,S^{-1} = \mathscr{H}_{\rm diag} = {\rm diag}\,(H_1, H_2)$

Canonical transformations preserve the form of Hamilton's equations in classical mechanics $i\frac{d}{dt}|\Psi\rangle = \mathscr{H}|\Psi\rangle$

Without loss of generality, the matrix S can be taken as unimodular, with $Det\ S = 1$

$$S = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}, \qquad c = \cos\,\zeta = \frac{1}{2}\left(e^{i\zeta} + e^{-i\zeta}\right), \quad s = \sin\,\zeta = \frac{1}{2i}\left(e^{i\zeta} - e^{-i\zeta}\right)$$

Analytical solution if B = constant, e. g. = 0 inside absorber $U = V - \Delta m \mp |\mu B|$

Initial condition at the entrance to absorber $\Psi(0) = \binom{1}{0}$

$$H_{1,2} = U_{1,2} - iW_{1,2} = \frac{1}{2}\left(U - iW \pm \sqrt{(U-iW)^2 + 4\epsilon^2}\right)$$

Eigenvalues are complex

$$\tan 2\zeta = \frac{2\epsilon}{U - iW} = \frac{2\epsilon(U+iW)}{U^2+W^2}$$

Rotation in Hilbert space for complex angle $\zeta = \theta + i\omega$

$$|\Psi(t)\rangle = \mathcal{S}(t)|\Psi(0)\rangle, \quad \mathcal{S}(t) = e^{-i\mathcal{H}t} = e^{-iS^{-1}\mathcal{H}_{\text{diag}}S\,t} = S^{-1}\text{diag}(e^{-iH_1t}, e^{-iH_2t})\,S$$

$$\mathcal{S}(t) = \begin{pmatrix} \mathcal{S}_{nn}(t) & \mathcal{S}_{nn'}(t) \\ \mathcal{S}_{n'n}(t) & \mathcal{S}_{n'n'}(t) \end{pmatrix} = \begin{pmatrix} c^2e^{-iH_1t} + s^2e^{-iH_2t} & cs(e^{-iH_1t} - e^{-iH_2t}) \\ cs(e^{-iH_1t} - e^{-iH_2t}) & s^2e^{-iH_1t} + c^2e^{-iH_2t} \end{pmatrix}$$

$$P_{nn}(t) = |S_{nn}(t)|^2 = |c|^4e^{-2W_1t} + |s|^4e^{-2W_2t} + 2\text{Re}(c^{*2}s^2e^{i\Delta Et})e^{-Wt}$$

$$P_{nn'}(t) = |S_{n'n}(t)|^2 = \frac{1}{4}|\sin 2\zeta|^2[e^{-2W_1t} + e^{-2W_2t} - 2\cos(\Delta Et)e^{-Wt}]$$

Zurab's formulas for passing the absorber from initial state $\Psi(0) = (1,0)^T$

After passing 3.5 mm Cd absorber

If no oscillations $\epsilon = \theta_0 = 0.$ $\quad P_{nn} = 7.937 \times 10^{-40}$

$\Delta m = 300\, neV, \theta_0 = 0.01 \quad P_{nn} = 2.351 \times 10^{-8}, P_{nn'} = 1.522 \times 10^{-4}$

$\Delta m = 300\, neV, \theta_0 = 0.001 \quad P_{nn} = 2.386 \times 10^{-12}, P_{nn'} = 1.544 \times 10^{-6}$

After passing 32 mm B_4C absorber

If no oscillations $\epsilon = \theta_0 = 0.$ $\quad P_{nn} = 2.06 \times 10^{-259}$

$\Delta m = 300\, neV, \theta_0 = 0.01 \quad P_{nn} = 4.586 \times 10^{-7}, P_{nn'} = 5.211 \times 10^{-4}$

$\Delta m = 300\, neV, \theta_0 = 0.001 \quad P_{nn} = 7.747 \times 10^{-11}, P_{nn'} = 8.778 \times 10^{-6}$

No previous oscillations in the vacuum before absorber

$P_{nn} = 7.937 \times 10^{-40}$

Exp(-3500/39)=1.06×10^{-39}

ρ − evolution calculation

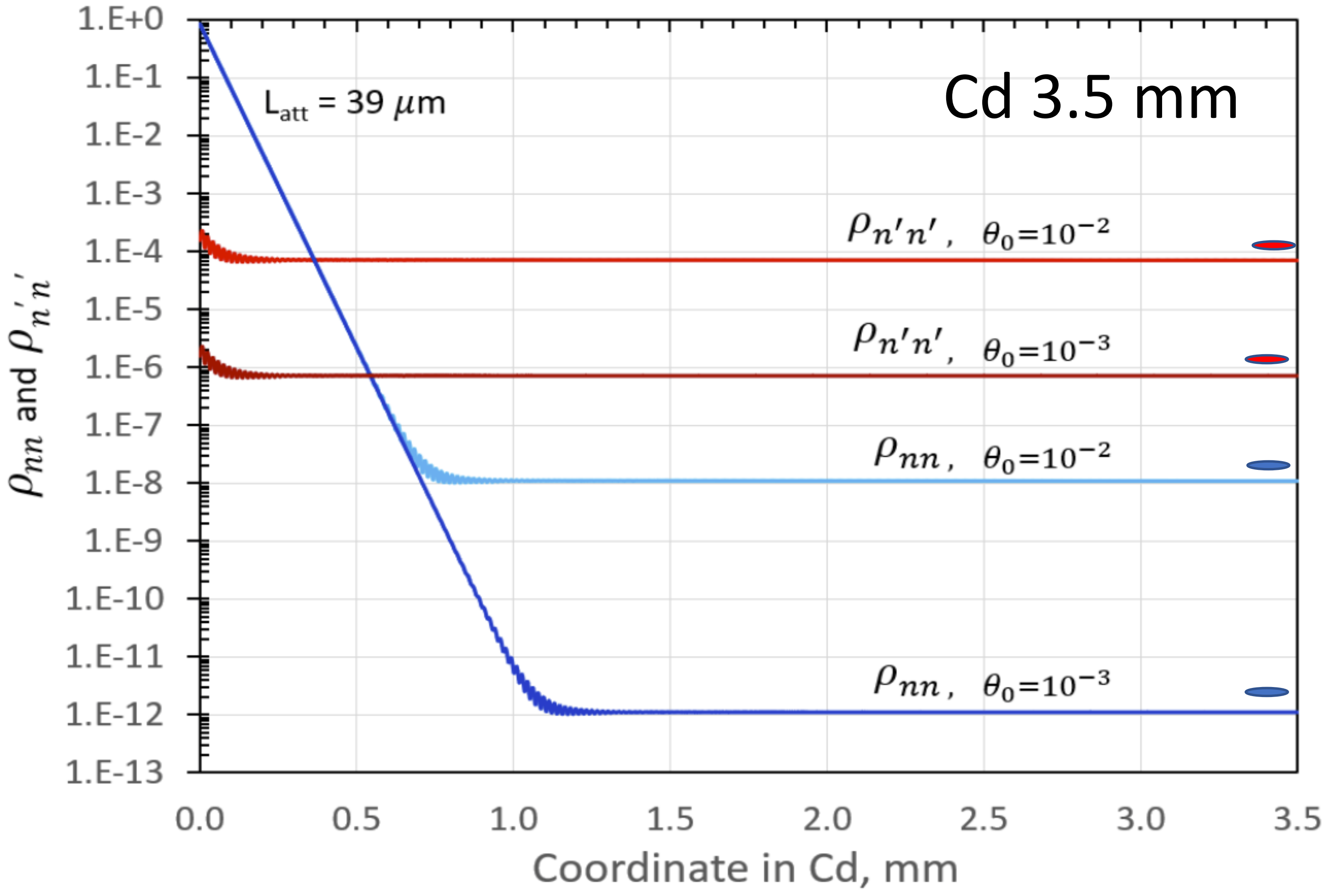
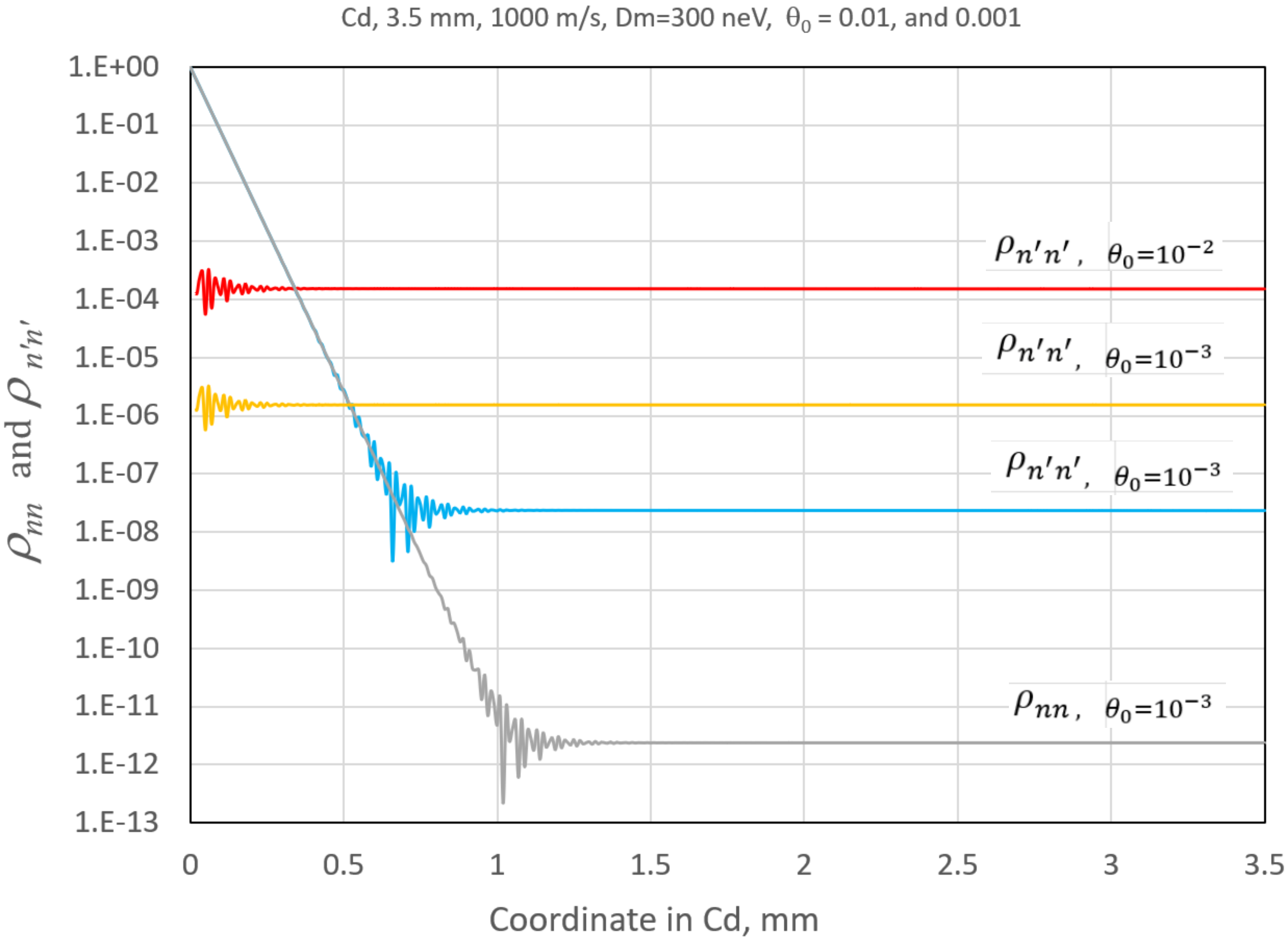
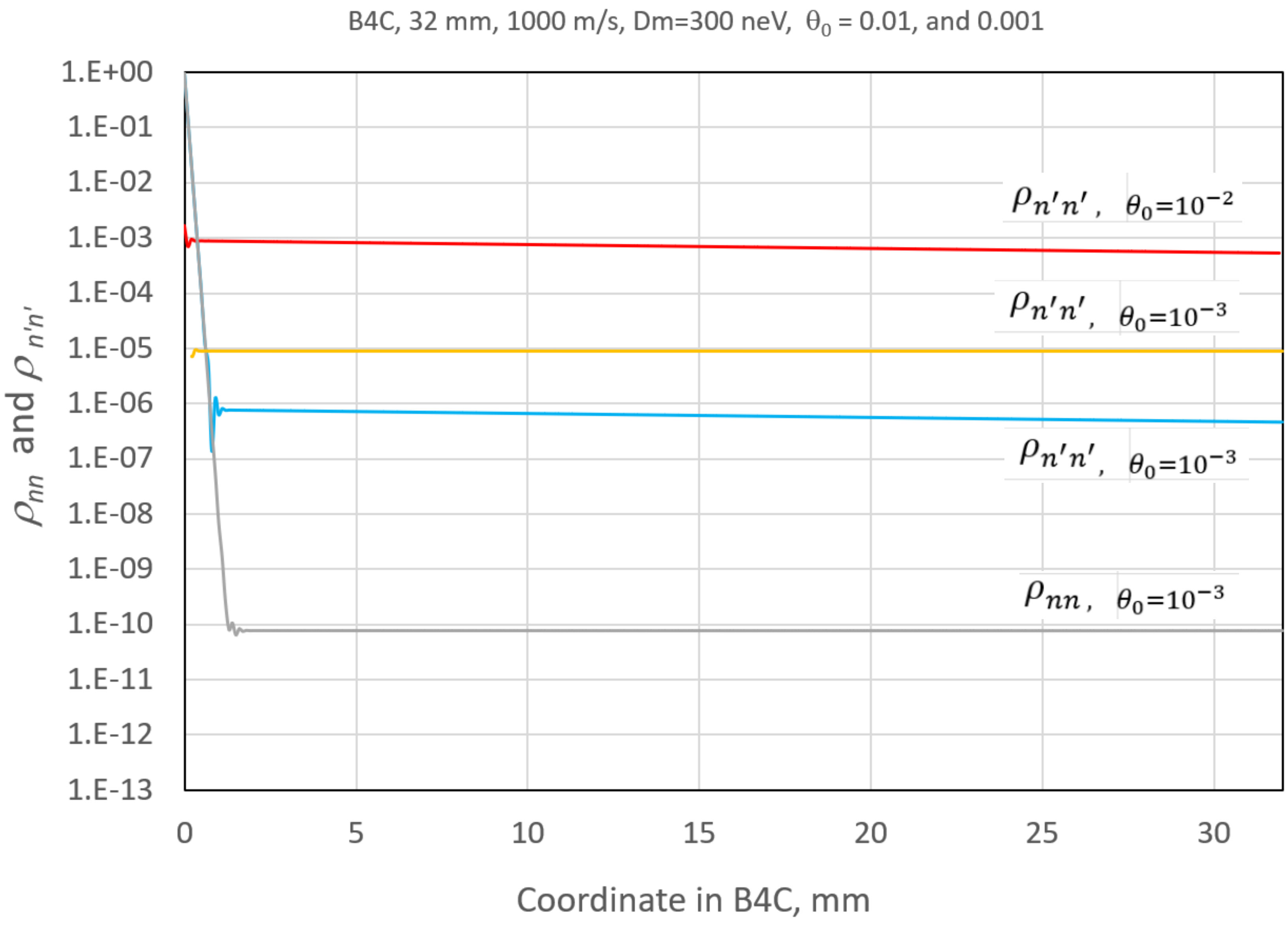


Figure 1. Evolution of (n, n') system in a 3.5 mm Cd absorber for $\Delta m = 300$ neV, $\theta_0 = 10^{-2}$ and 10^{-3}, and for velocity $v = 1000$ m/s. The averaged density matrix at the entrance of the Cd absorber after passing 3 m in air is used as an initial condition. Magnetic field is $B = 0$. Neutron components ρ_{nn} are shown in light/dark blue and mirror neutron component $\rho_{n'n'}$ in light/dark red colors.

Analytical solution (Zurab)



Analytical solution (Zurab)



Initial state: average density matrix $\rho(0)$ in vacuum

Initial state $\Psi(0) = (1,0)^T$

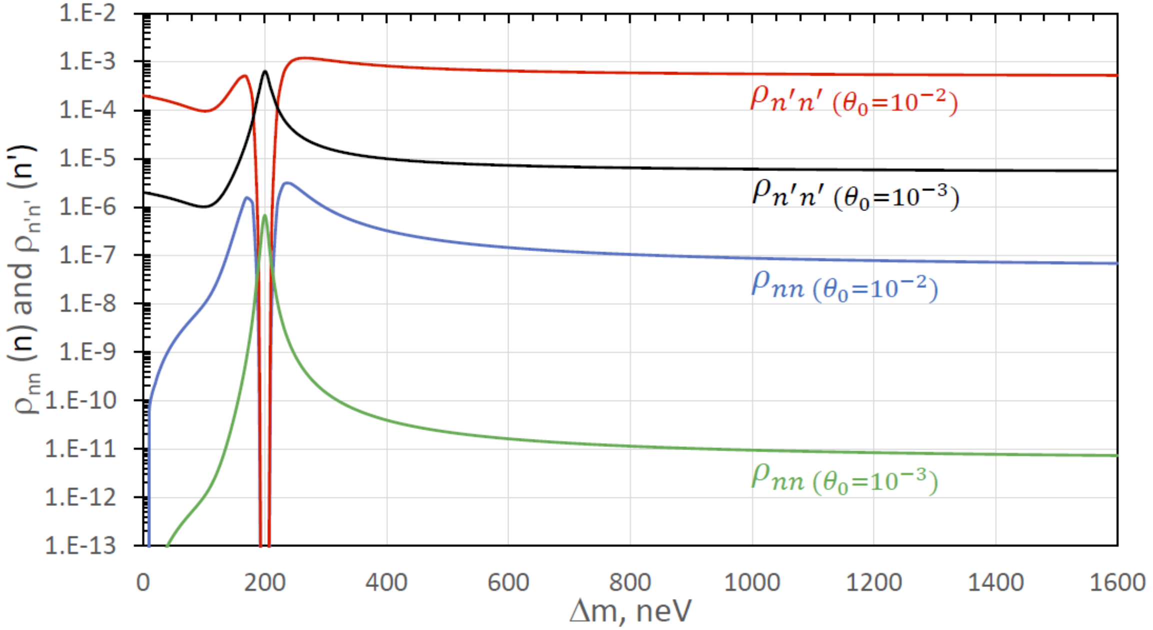
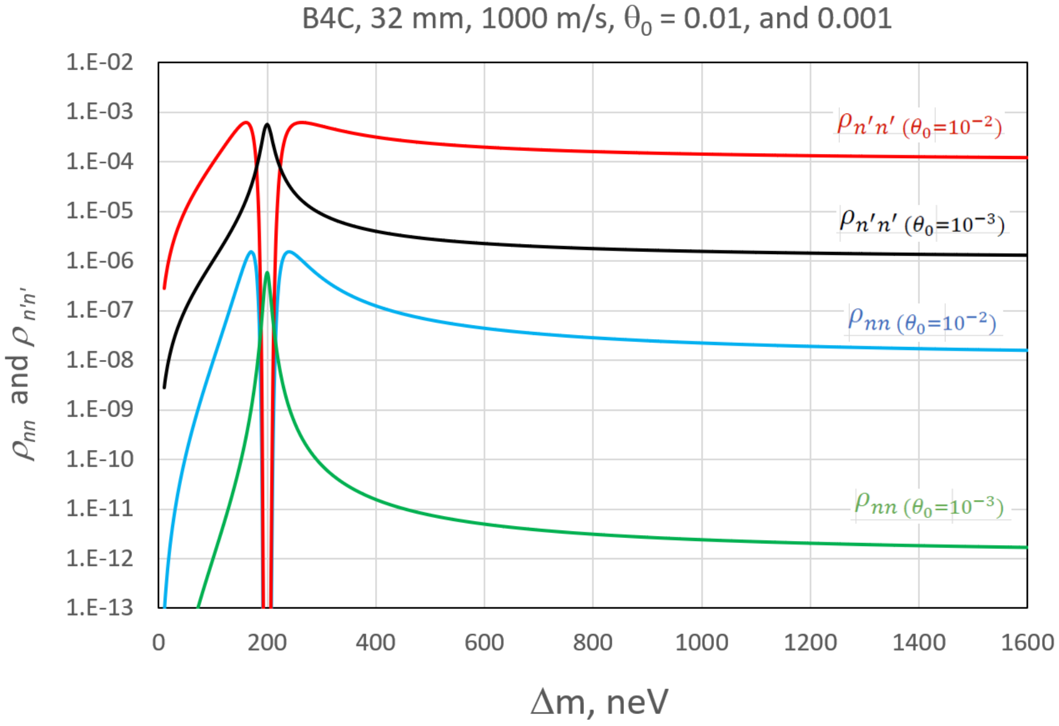


Figure 2. Δm dependence of probability ρ_{nn} and $\rho_{n'n'}$ for two values of angles $\theta_0 = 0.01$ and 0.001 after the neutron passes through the 32 mm B_4C absorber starting from the average density matrix in a vacuum. The value for the magnetic field $B = 0$.

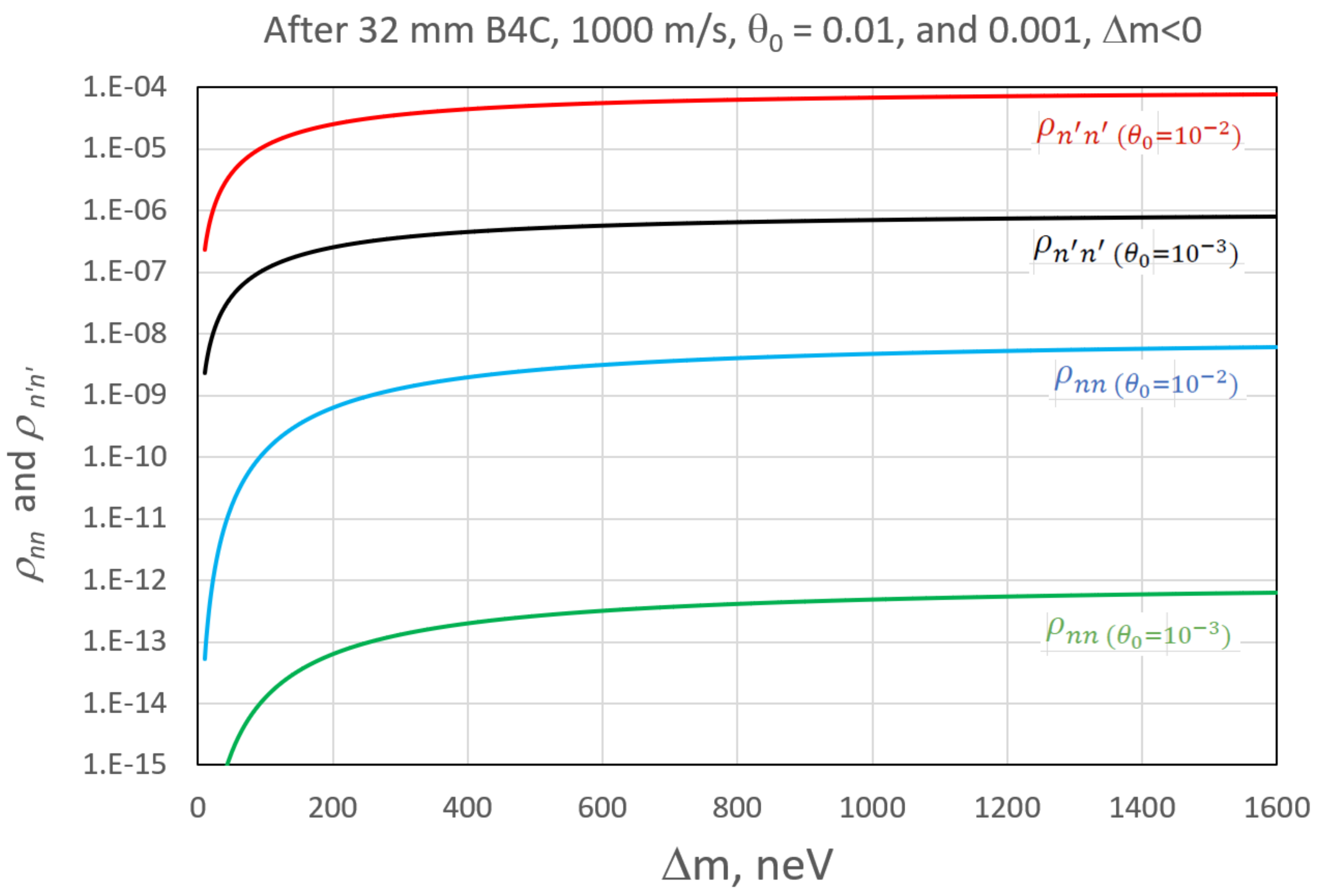
Density matrix evolution

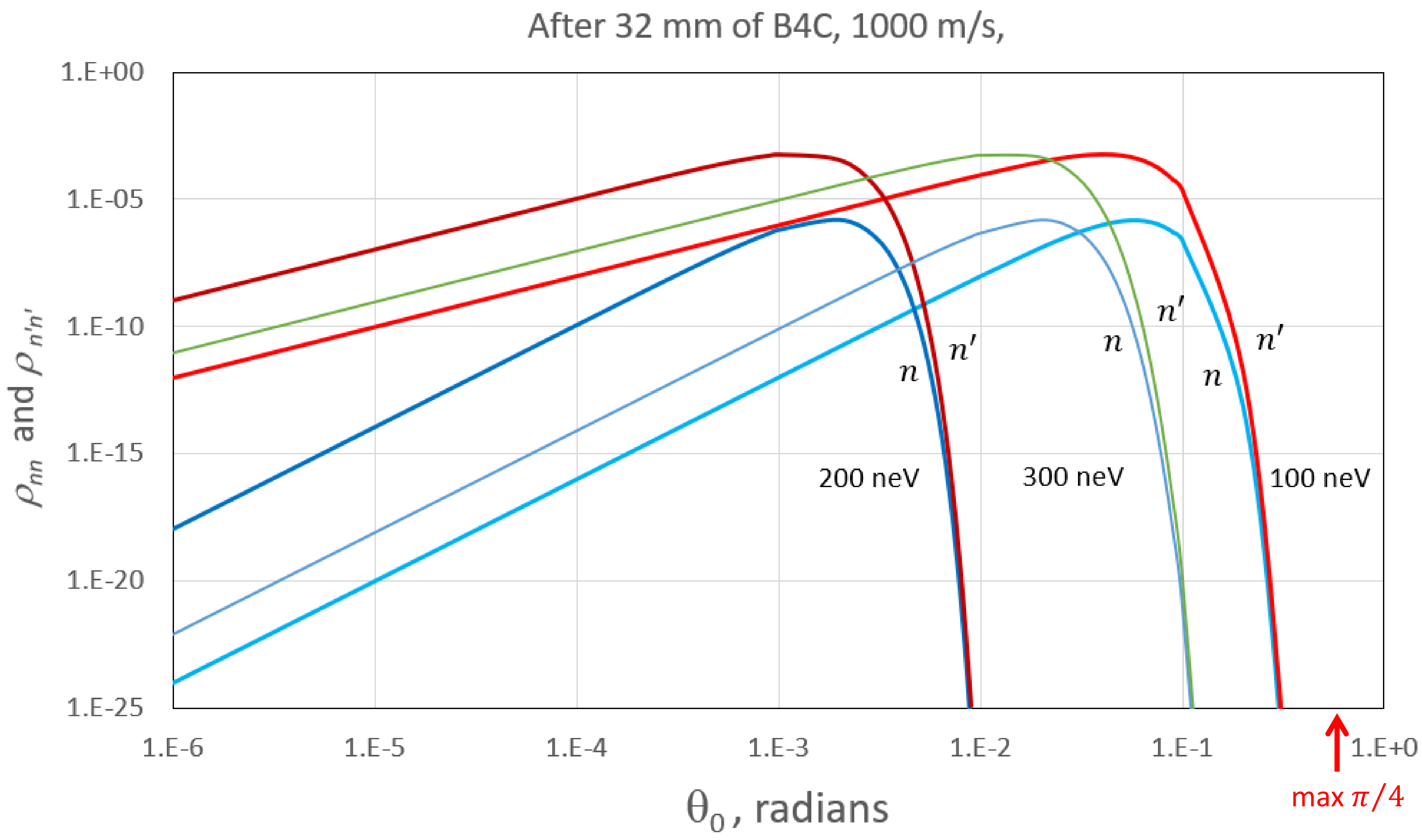


Analytical solution (Zurab)

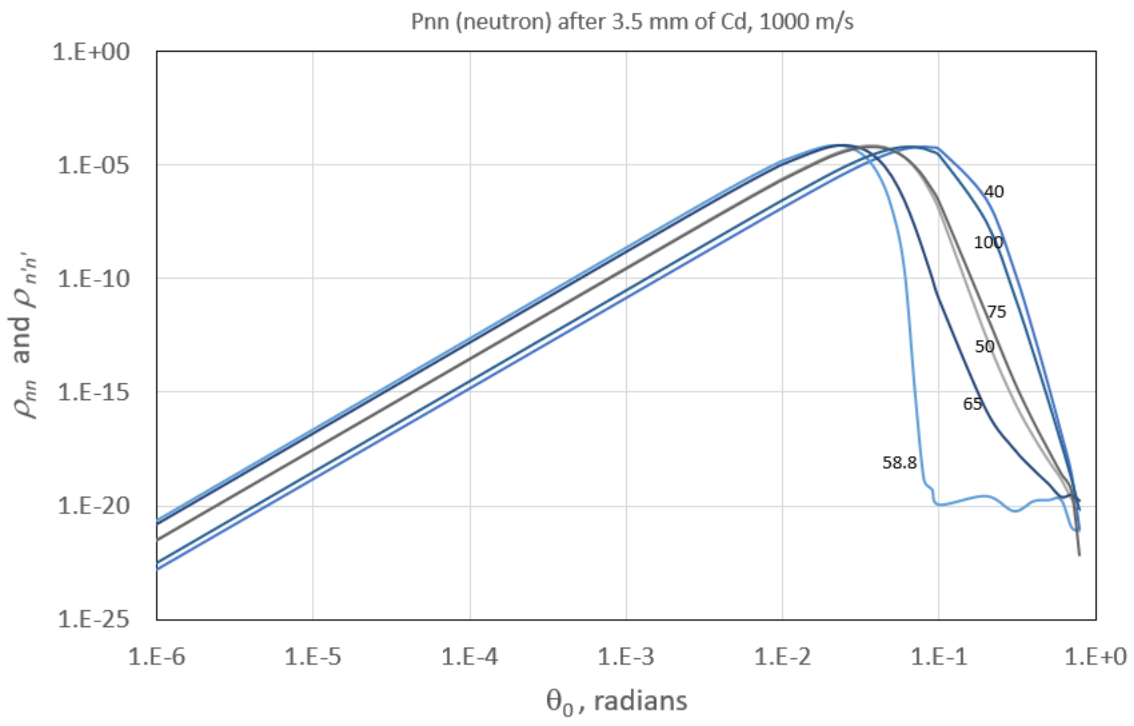
Analytical solution (Zurab) for $m_n > m_{n'}$

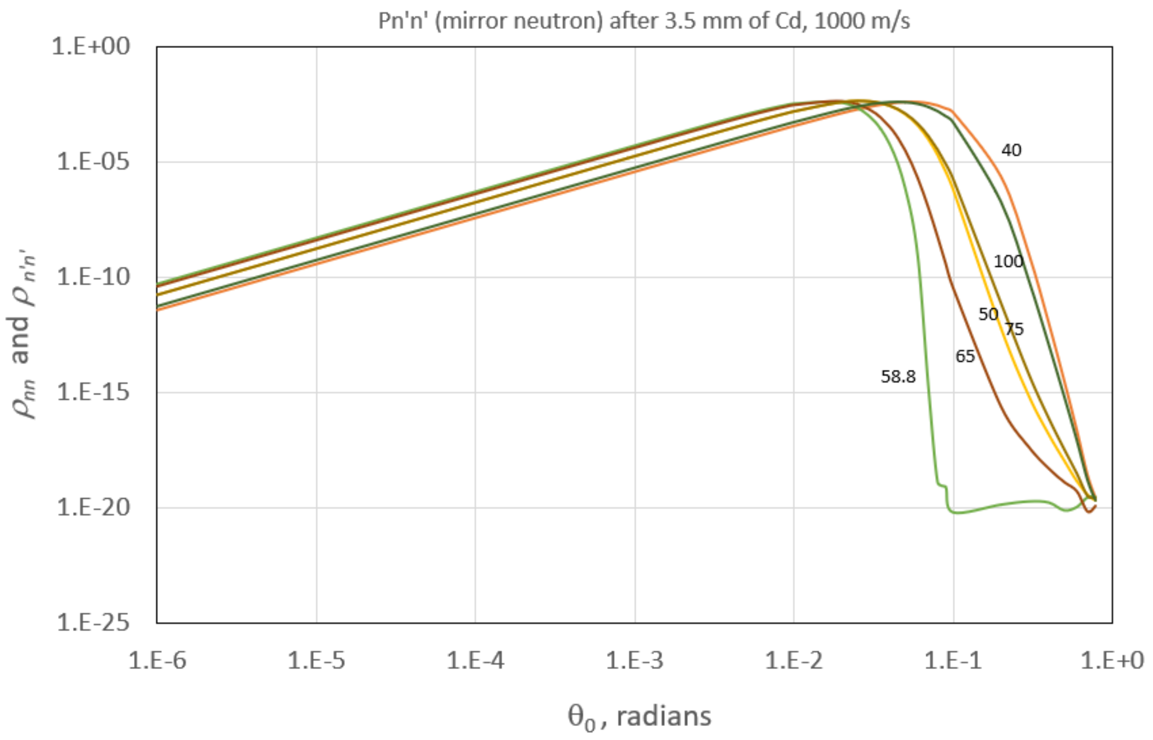
Initial state $\Psi(0) = (1,0)^T$




After 32 mm of B4C, 1000 m/s,
1.E+00
1.E-05
1.E-10
1.E-15
1.E-20
1.E-25
ρ_{nn} and $\rho_{n'n'}$
1.E-6
1.E-5
1.E-4
1.E-3
1.E-2
1.E-1
1.E+0
θ_0 , radians
max $\pi/4$
n
n'
n
n'
n
n'
200 neV
300 neV
100 neV

3.5 mm Cd , 1000 m/s





What is so far missing in analytical solution:

Need to use initial conditions for $\Psi(0)$ coming from the previous motion of the system in vacuum or in air

Like we used for the density matrix evolution:

$$\bar{\rho} = \begin{pmatrix} 1 - 2\theta_0^2 & -\theta_0 \\ -\theta_0 & 2\theta_0^2 \end{pmatrix}$$

$$\begin{aligned} \bar{\rho}_{nn} &= \left(1 - \tfrac{1}{2} \cdot \sin^2 2\theta_0\right) \cdot e^{-\Delta z / L_{air}} \\ \mathrm{Re}\bar{\rho}_{nn'} = \mathrm{Re}\bar{\rho}_{n'n} &= -\tfrac{1}{4} \cdot \sin 4\theta_0 \cdot e^{-\Delta z / L_{air}} \\ \mathrm{Im}\bar{\rho}_{nn'} = \mathrm{Im}\bar{\rho}_{n'n} &= 0 \\ \bar{\rho}_{n'n'} &= \tfrac{1}{2} \cdot \sin^2 2\theta_0 \cdot e^{-\Delta z / 2L_{air}} \end{aligned}$$

In vacuum for small θ_0

In air with absorption

- Meaning of the average density matrix is repeating itself $E[\bar{\rho}] = \bar{\rho}$ (makes sense for non-absorptive H)

Re/Im parts of some absorbers

Neutron scattering lengths and cross sections							
Isotope	conc	Coh b	Inc b	Coh xs	Inc xs	Scatt xs	Abs xs
H	---	-3.7390	---	1.7568	80.26	82.02	0.3326
1H	99.985	-3.7406	25.274	1.7583	80.27	82.03	0.3326
2H	0.015	6.671	4.04	5.592	2.05	7.64	0.000519
3H	(12.32 a)	4.792	-1.04	2.89	0.14	3.03	0
He	---	3.26(3)	---	1.34	0	1.34	0.00747
3He	0.00014	5.74-1.483*i*	-2.5+2.568*i*	4.42	1.6	6	5333.(7.)
B	---	5.30-0.213*i*	---	3.54	1.7	5.24	767.(8.)
10B	20	-0.1-1.066*i*	-4.7+1.231*i*	0.144	3	3.1	3835.(9.)
11B	80	6.65	-1.3	5.56	0.21	5.77	0.0055
C	---	6.6460	---	5.551	0.001	5.551	0.0035
Cd	---	4.87-0.70*i*	---	3.04	3.46	6.5	2520.(50.)
106Cd	1.25	5.(2.)	0	3.1	0	3.1(2.5)	1
108Cd	0.89	5.4	0	3.7	0	3.7	1.1
110Cd	12.51	5.9	0	4.4	0	4.4	11
111Cd	12.81	6.5	---	5.3	0.3	5.6	24
112Cd	24.13	6.4	0	5.1	0	5.1	2.2
113Cd	12.22	-8.0-5.73*i*	---	12.1	0.3	12.4	20600.(400.)
114Cd	28.72	7.5	0	7.1	0	7.1	0.34
116Cd	7.47	6.3	0	5	0	5	0.075
149Sm	13.9	-19.2-11.7*i*	(+/-)31.4-10.3*i*	63.5	137.(5.)	200.(5.)	42080.(400.)
Gd	---	6.5-13.82*i*	---	29.3	151.(2.)	180.(2.)	49700.(125.)
157Gd	15.7	-1.14-71.9*i*	(+/-)5.(5.)-55.8*i*	650.(4.)	394.(7.)	1044.(8.)	259000.(700.)