Quantum Simulation of QFT in the Front Form

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Motivation

The currently dominant approach to digital quantum simulation of QFT is based on the equal-time lattice formulation.

A lot of progress, a lot of open questions:

- Gauge symmetry protection — highly non-trivial.
- Difficult to extract information about observables.
- Qubit number / lattice size:

$$\mathtt{Q}_{QCD} \quad \underbrace{(\text{internal DOFs})}_{50} \quad \underbrace{L^{D \quad 1}}_{20^3} \quad 400,000 \text{ qubits.} \tag{1}$$

Can we overcome these difficulties by using some alternative approach?

Quantum Simulation in the Front Form

Good news:

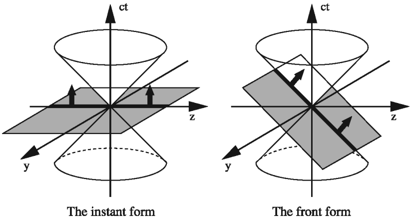
- Fact #1: Numerous techniques for the Digital Quantum Simulation of Quantum Chemistry have been developed in the last decades.
- Fact #2: QFT in the **light-front (LF)** [1] formalism looks much like non-relativistic many-body physics!

[1] Within this talk *'front-form'* *'light-front'* *'light-cone'*.

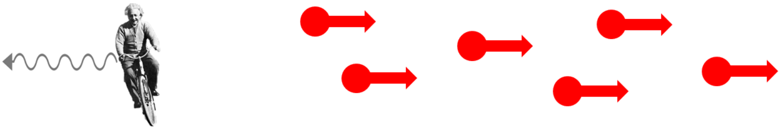
Quantum Field Theory in the Front Form

The "light-cone time" x^+
and "light-cone distance" x :

$$x \quad = x^0 \quad x^1 . \qquad (2)$$



From the point of view of a massless particle moving, say, to the **left**, all the massive particles move to the **right**:



All the light-cone momenta of massive particles are **positive**.

Why use the LF formulation?

	LF QFT features	Advantages for QC
Resources	No ghost fields Linear EoM	Low qubit count
	LF momentum > 0	Efficient encoding
Evolution	Sparse Hamiltonians	Using sparsity-based methods
Measurement	LF wavefunction ! ! static quantities; Simple form of operators in the second-quantized formalism	Simple form of measurement operators
Other	Trivial vacuum, fewer cut-offs, no fermion doubling, form invariance of H, equal treatment of matter and gauge fields in the $A^+ = 0$ gauge	

DLCQ: ϕ^4 in $1+1D$

Discretized Light-Cone Quantization (DLCQ)[2]

= Light-Cone Hamiltonian + Second Quantization

$$L = \frac{1}{2}(\partial\phi)^2 \quad \frac{1}{2}m_B^2\phi^2 \quad \frac{\lambda}{4!}\phi^4 \ ,$$

$$H = \sum_{n} \dots a_n^\dagger a_n + \sum_{klmn} \dots a_k^\dagger a_l^\dagger a_m a_n + \sum_{klmn} (\dots a_k^\dagger a_l a_m a_n + \text{c.c.}) \ ,$$

$$K = \sum_{n} n a_n^\dagger a_n \ .$$

We solve H in the basis of *Fock states* $\{|F\rangle\}$:

$$\{|F\rangle\} \text{ at } K=5: \quad |1^5\rangle,\ |1^2,3\rangle,\ |1,2^2\rangle,\ |1,4\rangle,\ |2,3\rangle\,. \qquad (3)$$

The number of $\{|F\rangle\}$ scales as $p(K) = O(\exp(\sqrt{K}))$.

The **lower bound** on the number of qubits is $\boxed{Q = O(\sqrt{K})}$.

[2] H.-C. Pauli, S.J. Brodsky, PRD **32**, 1985.
A. Harindranath., J.P. Vary, PRD **36**, 1987.

Encoding Fock states

Two ways of encoding a Fock state $|F\rangle = |n_1^{w_1}, n_2^{w_2}, \ldots\rangle$.

I. *Direct encoding* — qubits store w_j (qubit register per mode):

$$|\Psi\rangle = |\underbrace{0101}_{w_1}\underbrace{1001}_{w_2}\ldots\rangle\,, \tag{4}$$

$$\mathsf{Q}_{\text{Direct}} = O(K \log K)\,. \tag{5}$$

II. *Compact encoding* — qubits store n_j and w_j, only for $w_j > 0$:

$$|\Psi\rangle = |\overbrace{\underbrace{0111}_{n_1}\underbrace{0101}_{w_1}\underbrace{1100}_{n_2}\underbrace{1001}_{w_2}\ldots}^{\text{at most } O(\sqrt{K}) \text{ modes}}\rangle\,, \tag{6}$$

$$\mathsf{Q}_{\text{Compact}} = \boxed{O(\sqrt{K} \log K)}\,. \tag{7}$$

In the presence of transverse dimensions:

$$\mathsf{Q}_{\text{Direct}} = \widetilde{O}(K \Lambda_{\perp}^{d-1}) \text{ vs. } \mathsf{Q}_{\text{Compact}} = \widetilde{O}(K)\,. \tag{8}$$

Encoding Fock states

Should we always use compact mapping? No, because the choice of encoding restricts the choice of simulation algorithms.

	Trotter (product formulas)	Sparsity (more advanced)
Direct	✓	✓
Compact	✗	✓

Using compact mapping results in longer circuits.

Near-term ! Variational ! Direct+Trotter

Far-future ! Hamiltonian evolution ! Tight on gates?

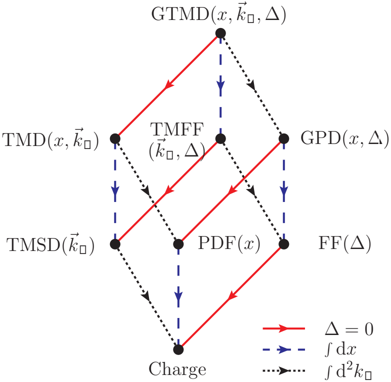
! { Yes ! Direct+Sparse
No ! Compact+Sparse

Measurement

Using QCs for simulating spectroscopy is particularly natural, as most of the LF observables have the form of

$$O = \text{poly}(a, a^y, b, b^y)\,, \qquad (9)$$

which can be easily measured in the quantum computer, once the final state is prepared.



(Pasquini, Lorce, 2012)

NISQ: VQE + BLFQ

In order to bring the computational requirements to the range of near-term devices, we shall use the Basis Light-Front Quantization (BLFQ) technique:

$$\text{BLFQ} = \text{Effective Light-Front Hamiltonian} + \text{Second Quantization} + \underline{\text{Smart Basis Choice}}$$

$$|F\rangle = |\xi_1^{w_1}, \xi_2^{w_2}, \dots\rangle \ , \tag{10}$$

where ξ_j denote solutions of some single-particle equation — not necessarily the plane waves!

Indeed, it makes sense to describe a confined system in the basis of solutions of a harmonic oscillator.

NISQ: VQE + BLFQ

How to extract the very essential information from QFT?

1. Restrict to the valence sector of the meson Fock space:

$$\boxed{|q\bar{q}\rangle}, |qq\bar{q}\bar{q}\rangle, |q\bar{q}g\rangle, |q\bar{q}gg\rangle, \ldots \tag{11}$$

2. Use relative momentum.
3. Use an effective interaction:[3]

$$H = H_0 + H_{\mathrm{NJL},\pi} = \overbrace{H_{\mathrm{transverse}}}^{2D \;\; \mathrm{HO}} + H_{\mathrm{longitudinal}} + H_{\mathrm{NJL},\pi} \,. \tag{12}$$

4. Use an efficient basis representation of the LF WF, the eigenbasis of H_0:
   - The spectrum H_0 can be found analytically.
   - H_0 already incorporates confinement.
   - $H_{\mathsf{transverse}}$ stems from AdS/QCD and corresponds to the linear confinement in equal time.

[3] Jia et al., `arXiv:1811.08512`.

NISQ: VQE + BLFQ

We write the second-quantized quark Hamiltonian as:

$$H = H_1 + H_2 + \dots \ , \tag{13}$$

where

$$\boxed{H_1 = \sum_{i,j} h_{ij} b_i^\dagger b_j} \ , \quad H_2 = \sum_{i,j,k,l} h_{ijkl} b_i^\dagger b_j^\dagger b_k b_l \ . \tag{14}$$

For the minimal cutoffs, spec $h_{ij} = \{139.6^2; 722.2^2; 827.8^2; 864.7^2\}$, with the two lowest eigenvalues corresponding to the masses of π and ρ mesons.

VQE + BLFQ

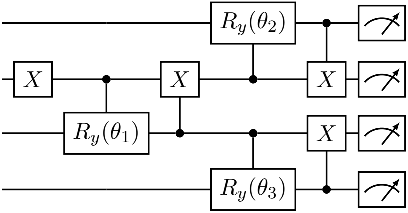
Direct mapping, state:

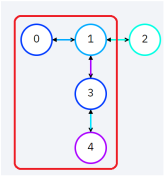
$$j\psi(\vec{\theta})i = \alpha_1 j0001i + \alpha_2 j0010i + \alpha_3 j0100i + \alpha_4 j1000i \ . \tag{15}$$

Multi-qubit Hamiltonian:

$$\begin{aligned} H = {} & 87397(IXXI + IYYI) \quad 53725(YZZY + XZZX) \\ & 320161(IIIZ + ZIII) \quad 173353(IZII + IIZI) \\ & + 69936(IIYY + IIXX + YZYI + XZXI \\ & IYZY \quad IXZX \quad YYII \quad XXII) + 987031IIII \ . \end{aligned} \tag{16}$$

Ansatz circuit (the angles $f\theta_1, \theta_2, \theta_3 g$ are the VQE parameters):





VQE + BLFQ

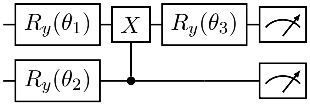
Compact mapping, state:

$$|\psi(\vec{\alpha})\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle \ . \qquad (17)$$

Multi-qubit Hamiltonian:

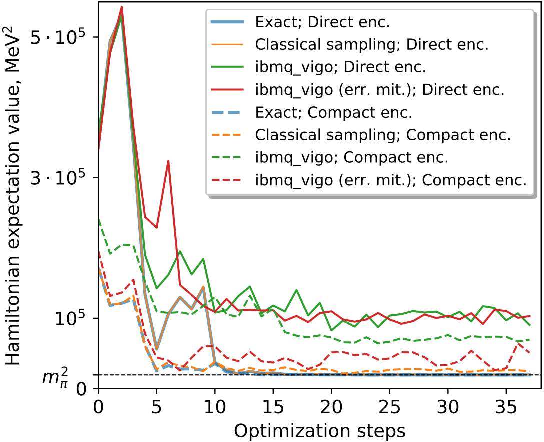
$$\begin{aligned} H = {} & 33671XX + 141122YY + 146807ZZ \\ & + 493515II + 139872(ZX - XZ) \ . \end{aligned} \qquad (18)$$

Ansatz circuit (the angles $\{\theta_1, \theta_2, \theta_3\}$ are the VQE parameters):



VQE + BLFQ

VQE minimization on `ibmq_vigo`, 8192 samples per term:



VQE improvements and BLFQ variants

General / vanilla VQE setting:

- $H = \text{poly}(a, a^\dagger, b, b^\dagger)$.
- Direct mapping only, because:
- Ansatz: Unitary Coupled Cluster.
- Various observables can be calculated efficiently.

VQE enhancements:

- Pauli term reduction.
- Contextual subspace VQE.
- Sparse measurements.
- Tapering off qubits.
- Extrapolation techniques.

BLFQ variants:

- Various interactions:
  Phenomenological (e.g. NJL)
  #
  Effective (e.g. one g exchange)
  #
  Dynamical gluons (QCD).
- Various basis choices: 3DHO, 2DHO + plane waves, etc.

Key Takeaways

- Numerous advantages of the second-quantized LF Hamiltonian formulation come in handy at the stage of quantum simulation.
- Various LF models (phenomenology, ab initio) and quantum simulation algorithms (heuristic, Hamiltonian evolution) can be employed, depending on available resources.
- Results:
  - 2002.04016 — adiabatic preparation of interacting eigenstates. Qubit counts and observables for Yukawa_{1+1} and QCD_{3+1}.
  - 2105.10941 — details of sparsity-based simulation in the compact encoding.
  - 2011.13443, 2009.07885 — variational algorithms, unitary coupled cluster, BLFQ-NJL model of light mesons.
- Several approaches to the simulation of scattering are currently under development.

Thank YOU!!