Non-Boolean Quantum Amplitude Amplification and ML applications

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Machine Learning

A high-level view of Machine Learning:

1. Design a highly parameterized machine to perform a task. (architecture, trainable weights)
2. Design a way to evaluate the performance of the machine (cost function).
   Often evaluated by averaging a loss function over the training data.
3. Train the machine by optimizing the cost function.

In typical hybrid-quantum-classical ML approaches

- (1) is done with quantum circuits parameterized by classical parameters
- (2) and (3) are performed classically

Can we do better in the post-NISQ era?

In this talk, I will lay the foundations for an inherently quantum technique for ML training.
Introduction: Boolean Amplitude Amplification

Given:
- A quantum system with basis states $|0\rangle, |1\rangle, \ldots, |N-1\rangle$
- A Boolean function $f: \{0, \ldots, N-1\} \rightarrow \{0,1\}$ ("bad"/"good")
- An initial superposition of the states $|\psi_0\rangle = A_0|0\rangle$
- An oracle for the function $U_f |x\rangle = (-1)^{f(x)} |x\rangle$

Goal: Amplify the "good" states

$$|\psi_0\rangle = \cos(\theta/2) |\psi_{bad}\rangle + \sin(\theta/2) |\psi_{good}\rangle$$

$|\psi_{bad}\rangle$ and $|\psi_{good}\rangle$ are the good and bad projections of $|\psi_0\rangle$, normalized to 1.

Amplitude Amplification: Apply "$S U_f"$ repeatedly on $|\psi_0\rangle$. After $k$ steps

$$|\psi_k\rangle = \cos(k\theta + \theta/2) |\psi_{bad}\rangle + \sin(k\theta + \theta/2) |\psi_{good}\rangle$$

We could use this to optimize/train a machine.

But the performance measure of machines is unlikely to be Boolean!
Non-Boolean Amplitude Amplification

Let’s make the function Non-Boolean:

$$\varphi: \{0, \ldots, N - 1\} \rightarrow \mathbb{R}
\quad U_\varphi |x\rangle = e^{i\varphi(x)} |x\rangle$$

“Goodness” of a state is captured by $\cos \varphi(x)$.

Goal: Preferentially amplify states, based on their goodness.

The original amplitude algorithm will not work in this case.

Non-Boolean Amplitude Amplification in Action

A toy example

\[ \varphi(x) = \frac{x \pi}{255 \frac{\pi}{4}}, \quad \text{for } x = 0, 1, \ldots, 255 \]

\[ U_{\varphi} |x\rangle = e^{i\varphi(x)} |x\rangle \]

Amplified state measurement probabilities after 1, 2, and 3 iterations:

![Graph showing amplified state measurement probabilities after 1, 2, and 3 iterations.](image)
Quantum Mean Estimation Algorithm

Goal: Estimate \( \langle \psi_0 | U_\varphi | \psi_0 \rangle \). Let \( \cos \theta = Re \langle \psi_0 | U_\varphi | \psi_0 \rangle \)

\[
|+, \psi_0 \rangle = \frac{|\eta_+\rangle + |\eta_-\rangle}{\sqrt{2}}
\]

\[
Q_{\text{iter}} |+, \psi_0 \rangle = \frac{e^{i\theta}|\eta_+\rangle + e^{-i\theta}|\eta_-\rangle}{\sqrt{2}}
\]

Quantum phase estimation can be used with \( Q_{\text{iter}} \) as the operator, and \( |+, \psi_0 \rangle \) as the state to estimate \( \theta \).

Toy example results:
Quantum Mean Estimation Algorithm

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\]

1. Quadratic speed-up over shot-based estimation.
2. Can be used in ML, for example to average over training data.
Back to Machine Learning

Training a machine:

\[
\text{Optimize} \left[ \overline{\text{Loss for one data sample}} \right]
\]

Non-Boolean amplification

Operator use in Quantum Mean Estimation

Accomplished in this circuit:
Final piece of the puzzle

Qaracterized circuits: Quantum circuits parameterized by qubits
Example 1: Classifier parameterized by $|x_U\rangle$

- 4th register: superposition of all training datasamples
- $U(x_U)$: output of classifier CNOT-ed with the target
- $\theta$ captures the average classification accuracy of the classifier.

Training can be done using amplitude amplification!
Example 2: State preparation circuit parameterized by $|x_A\rangle$

- **4th register**: $|0\rangle$
- $U$: Evaluates the prepared state.
- This circuit produces an evaluation oracle for $|x_A\rangle$.
- Training can be done using amplitude amplification!
Results from QHACK 2021

Work done in collaboration with Evan Peters, University of Waterloo & FQI

Presentation at [https://peterse.github.io/groveropt/](https://peterse.github.io/groveropt/)

Parameterizing circuits with qubits aka Qarameterized circuits:
Results from QHACK 2021

Left panel: Cost function
Right panel: Sampling probability of the parameters, post training using non-Boolean amplification

Thank you!
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