Stopped-pion neutrino scattering off ^{127}I and ^{133}Cs using a hybrid approach of shell model and MQPM

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May 21, 2022

Background

- Experimental detection of neutrinos relies mostly on their scattering off atomic nuclei in detectors.
- Cesium-Iodide based neutrino detectors have been and are in use among a number of experiments around the world.
- The neutrino scattering properties of the stable isotopes ^{127}I and ^{133}Cs are therefore of notable interest.

General considerations

- The goals of the research were to obtain realistic inelastic scattering cross sections for stopped-pion neutrinos off the nuclei ^{127}I and ^{133}Cs along with nuclear de-excitation data using a Monte Carlo simulation.
- The plan was to utilize a hybrid approach of using shell model and Microscopic quasiparticle-phonon model (MQPM) for the cross sections and de-excitation data, along with just MQPM for cross sections.

Motivation for a hybrid approach

- MQPM produces realistic spectra in reasonable times with the upper end of the spectra reaching energies as high as 25 MeV and above.
- The agreement with experiment at the low-energy end typically has clear room for improvement.
- The idea was to use a hybrid model for where the low-energy ends of the spectra are calculated with the shell model up to a cutoff energy (3 MeV), and for the rest of the spectra MQPM results are used.

Model of Argon Reaction Low Energy Yields

- Model of Argon Reaction Low Energy Yields (MARLEY) is a Monte Carlo event generator for neutrino-nucleus interactions at energies of tens-of-MeV and below, developed by Steven Gardiner.
- MARLEY is currently limited to employ the allowed approximation.
- For ^{127}I and ^{133}Cs with ground state spin-parities $5/2^+$ and $7/2^+$, this limited the possible final nuclear states after a scattering event to $3/2^+$, $5/2^+$, $7/2^+$ and $5/2^+$, $7/2^+$, $9/2^+$ respectively.

Inputs for MARLEY

- MARLEY simulates neutrino-nucleus scattering events and requires the energies and $B(GT_0)$ strengths for each state as inputs.
- The $B(GT_0)$ strengths

$$B(GT_0) = \frac{g_A^2}{2J_i+1} \left| \langle f || \sum_k \boldsymbol{\sigma}(k) t_0(k) || i \rangle \right|^2 \tag{1}$$

were obtained through an approximate conversion

$$B(GT_0) = \frac{4\pi g_A^2}{3\mu_V^2} \frac{B(M1)}{\mu_N^2} = 0.308 \frac{B(M1)}{\mu_N^2}, \tag{2}$$

with the $B(M1)$ strengths obtained using shell model and MQPM.

- Experimental energies were also used for some of the lowest states for both nuclei.

Pure MQPM results

- MARLEY has the limitation that only allowed transitions can be modeled.
- The contributions from forbidden transitions are important and their magnitude should be determined.
- To this end, the folded cross sections for the pure MQPM spectra were computed.

Neutrino spectra

- The neutrinos considered in this work those produce in stopped-pion decay (ν_e, ν_μ and $\overline{\nu}_\mu$).
- The normalized distributions used for them were:

$$
\begin{aligned}
&p_{\nu_e}(E_{\nu_e}) = 96\frac{E_{\nu_e}^2}{m_\mu^4}(m_\mu - 2E_{\nu_e}), \\
&\nu_\mu \text{ are monoenergetic with an energy of } 29.8 \text{ MeV}, \\
&p_{\overline{\nu}_\mu}(E_{\overline{\nu}_\mu}) = 16\frac{E_{\overline{\nu}_\mu}^2}{m_\mu^4}(3m_\mu - 4E_{\overline{\nu}_\mu}),
\end{aligned}
\qquad (3)
$$

  where $E_{\nu_{\overline{\mu}}}$ and E_{ν_e} range from 0 to $m_\mu/2$.
- Folded scattering cross sections for all flavours of neutrinos produced in were computed using MARLEY.

On the nuclear structure

- MQPM states are constructed by coupling BCS-quasiparticles with quasiparticle random-phase approximation (QRPA) phonons.
- QRPA phonons in turn are constructed by pairs of BCS-quasiparticles coupled together.
- It is therefore necessary to first solve the BCS and QRPA equations for an even-even nucleus adjacent to the odd MQPM nucleus of interest to obtain the quasiparticles and phonons.

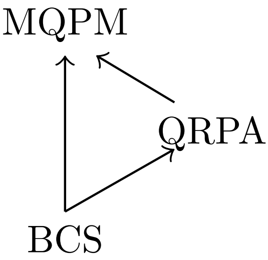


Figure: The hierarchy of the applied nuclear models.

The BCS model part 1

Given a set of single-particle orbitals $|\alpha\rangle$ and the corresponding creation operators $c_\alpha^\dagger$, the nuclear BCS state is defined according to

$$|\mathrm{BCS}\rangle = \prod_{\alpha>0} (u_a - v_a c_\alpha^\dagger \tilde{c}_\alpha^\dagger) |\mathrm{CORE}\rangle . \quad (4)$$

The BCS quasiparticle creation and annihilation operators are defined according to the Bogoliubov-Valatin transform

$$\begin{aligned} a_\alpha^\dagger &= u_a c_\alpha^\dagger + v_a \tilde{c}_\alpha, \\ \tilde{a}_\alpha &= u_a \tilde{c}_\alpha + v_a c_\alpha^\dagger. \end{aligned} \quad (5)$$

These operators fulfil the same commutation relations amongst themselves as the corresponding particle operators, and $\tilde{a}_\alpha |\mathrm{BCS}\rangle = 0$.

The BCS model part 2

The BCS equations can then be derived from the constrained variational problem

$$\delta \langle \mathrm{BCS}| \hat{\mathcal{H}} |\mathrm{BCS}\rangle = 0, \tag{6}$$

where

$$\hat{\mathcal{H}} \equiv \hat{H} - \lambda_{\mathrm{p}} \hat{n}_{\mathrm{p}} - \lambda_{\mathrm{n}} \hat{n}_{\mathrm{n}}. \tag{7}$$

The BCS state is not an eigenstate of the particle number, and the average particle numbers for both species of nucleons were constrained to match their actual numbers by introducing the Lagrange multipliers $\lambda_{\mathrm{p/n}}$ in the variational problem. After derivation, the BCS equations can be solved iteratively to obtain the occupation amplitudes u_a and v_a.

QRPA part 1

QRPA is a nuclear model where excited states of an even-even nucleus are constructed by coupling BCS quasiparticles to good angular momentum according to

$$Q^{\dagger}_{\omega\equiv nJ^{\pi}M} = \sum_{ab} \left[X^{\omega}_{ab} \mathcal{N}_{ab}(J) \left[a^{\dagger}_{a} a^{\dagger}_{b} \right]_{JM} + Y^{\omega}_{ab} \mathcal{N}_{ab}(J) \left[\tilde{a}_{a} \tilde{a}_{b} \right]_{JM} \right], \tag{8}$$

where

$$\mathcal{N}_{ab}(J) \equiv \frac{\sqrt{1+\delta_{ab}(-1)^{J}}}{1+\delta_{ab}}. \tag{9}$$

The excited states $|\omega\rangle$ are then generated by operating on the QRPA vacuum

$$|\omega\rangle = Q^{\dagger}_{\omega} |\mathrm{QRPA}\rangle, \quad \text{where} \quad Q_{\omega} |\mathrm{QRPA}\rangle = 0. \tag{10}$$

QRPA part 2

The QRPA equations written in matrix form are

$$\begin{pmatrix} \mathsf{A} & \mathsf{B} \\ -\mathsf{B}^* & -\mathsf{A}^* \end{pmatrix} \begin{pmatrix} X^\omega \\ Y^\omega \end{pmatrix} = E_\omega \begin{pmatrix} X^\omega \\ Y^\omega \end{pmatrix} . \qquad (11)$$

Here A is the QTDA matrix and B is the correlation matrix. The equations can be solved for the amplitudes X^ω_{ab} and Y^ω_{ab}. The QRPA vacuum is a correlated ground state of the form

$$|\mathrm{QRPA}\rangle = \mathcal{N}_0 e^S |\mathrm{BCS}\rangle , \qquad (12)$$

where $\mathcal{N}_0$ is a normalization constant and

$$S = \frac{1}{2} \sum_{JM} \sum_{a \le b, c \le d} C_{abcd}(J) \mathcal{N}_{ab}(J) \left[a^\dagger_a a^\dagger_b \right]_{JM} \mathcal{N}_{ab}(J) \left[\tilde{a}^\dagger_a \tilde{a}^\dagger_b \right]_{JM} , \qquad (13)$$

with the coefficients $C_{abcd}(J)$ chosen so that $Q_\omega |\mathrm{QRPA}\rangle = 0$ is fulfilled.

MQPM

MQPM can be used to model the states of an odd nucleus by coupling the BCS quasiparticles and QRPA phonons to form creation operators

$$\Gamma_i^\dagger(JM) = \sum_{n_b} C^i_{n_b} a^\dagger_{b=n_b JM} + \sum_a D^i_{a\omega} \left[a^\dagger_a Q^\dagger_\omega \right]_{JM} . \qquad (14)$$

MQPM states thus consist of quasiparticle and quasiparticle-phonon components. The amplitudes can be solved from the MQPM equations

$$\begin{pmatrix} A & B \\ B^{\mathrm{T}} & A' \end{pmatrix} \begin{pmatrix} C^i \\ D^i \end{pmatrix} = \Omega_i \begin{pmatrix} 1 & 0 \\ 0 & N \end{pmatrix} \begin{pmatrix} C^i \\ D^i \end{pmatrix} . \qquad (15)$$

Applying the nuclear models

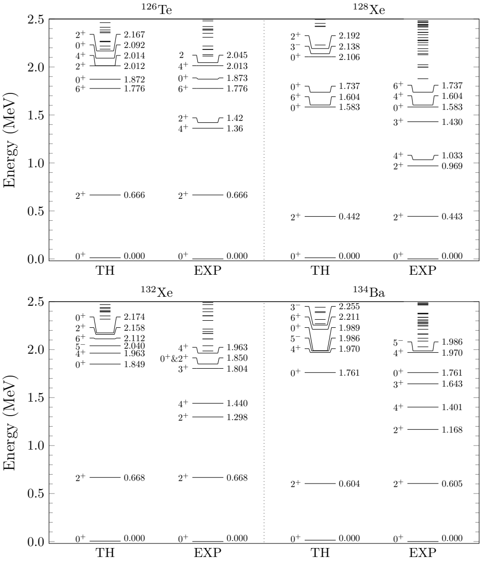
- To model the nuclei ^{127}I and ^{133}Cs with MQPM, the BCS and QRPA calculations were first done for the adjacent even-even reference nuclei ^{126}Te, ^{128}Xe, ^{132}Xe and ^{134}Ba.
- In the BCS calculations, the lowest quasiparticle energies were fit to the experimental values defined in terms of the nucleon separation energies as

$$\Delta_{\mathrm{p}}(A,Z) = \frac{(-1)^{Z+1}}{4}[S_{\mathrm{p}}(A+1,Z+1) - 2S_{\mathrm{p}}(A,Z) + S_{\mathrm{p}}(A-1,Z-1)], \tag{16}$$

$$\Delta_{\mathrm{n}}(A,Z) = \frac{(-1)^{A-Z+1}}{4}[S_{\mathrm{n}}(A+1,Z) - 2S_{\mathrm{n}}(A,Z) + S_{\mathrm{n}}(A-1,Z)]. \tag{17}$$

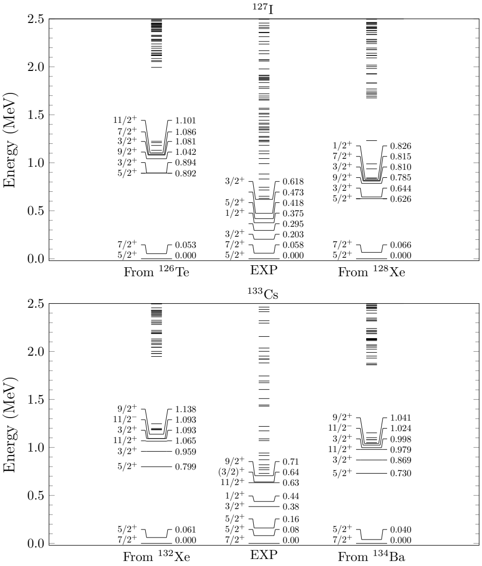
QRPA spectra

The theoretical QRPA energies of the lowest state for each spin-parity were also fitted to the corresponding experimental values. The results were generally good, and they are presented with the corresponding experimental spectra in the figure on the right.



MQPM spectra

All the fitting to experimental data was done at the BCS and QRPA calculations. The MQPM calculations were ran with the parameters obtained from the previous models. The results were fair, but there are differences with the corresponding experimental spectra, particularly at the low-energy end.



Folded cross sections

Figure: The folded scattering cross sections for all neutrino flavours, obtained with the MQPM results. Results are presented for both odd nuclei using all even-even reference nuclei. In units of 10^{-41} cm^2.

	^{127}I		^{133}Cs	
ν flavor	From ^{126}Te	From ^{128}Xe	From ^{132}Xe	From ^{134}Ba
e	10.24	10.23	10.12	9.26
μ	6.54	6.33	6.35	5.70
$\overline{\mu}$	12.23	12.32	12.29	11.32

Figure: The folded scattering cross sections for all neutrino flavours, obtained with MARLEY using the hybrid model. Results are presented for both odd nuclei using all even-even reference nuclei. In units of 10^{-41} cm^2.

	^{127}I		^{133}Cs	
ν flavor	From ^{126}Te	From ^{128}Xe	From ^{132}Xe	From ^{134}Ba
e	12.639	11.650	10.083	8.8056
μ	8.992	8.206	7.141	6.248
$\overline{\mu}$	17.949	16.623	14.364	12.529

Discrepancies between the results

- The MARLEY results are mostly higher than the MQPM results despite having fewer states in the calculations.
- The approximate conversion isn't applicable for the nuclei considered here. Instead, the presence of the orbital and isoscalar parts of the full $M1$ operator

$$M1 = \sum_k \left[\boldsymbol{l}(k) t_0(k) + (\mu_{\mathrm{S}} - 0.5) \frac{\boldsymbol{\sigma}(k)}{2} + \mu_{\mathrm{V}} \boldsymbol{\sigma}(k) t_0(k) \right] \tag{18}$$

need to be taken into account properly.

Conclusions for the pure MQPM results

Figure: The folded scattering cross sections for all neutrino flavours, obtained with the MQPM results. Results are presented for both odd nuclei using all even-even reference nuclei. In units of 10^{-41} cm^2.

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- There is little variance between the pure MQPM results for the two nuclei for all neutrino flavors.
- The differences in results from different reference nuclei are all also relatively small.

Plans for the future

- Computing the $B(GT_0)$ strengths exactly without the approximate conversion.
- Obtaining accurate cross sections with MARLEY.
- Analyzing the de-excitation data produced by MARLEY.

End

Thank you for listening.