ORNL Student Symposium

Collective Neutrino Oscillations on a Quantum Computer

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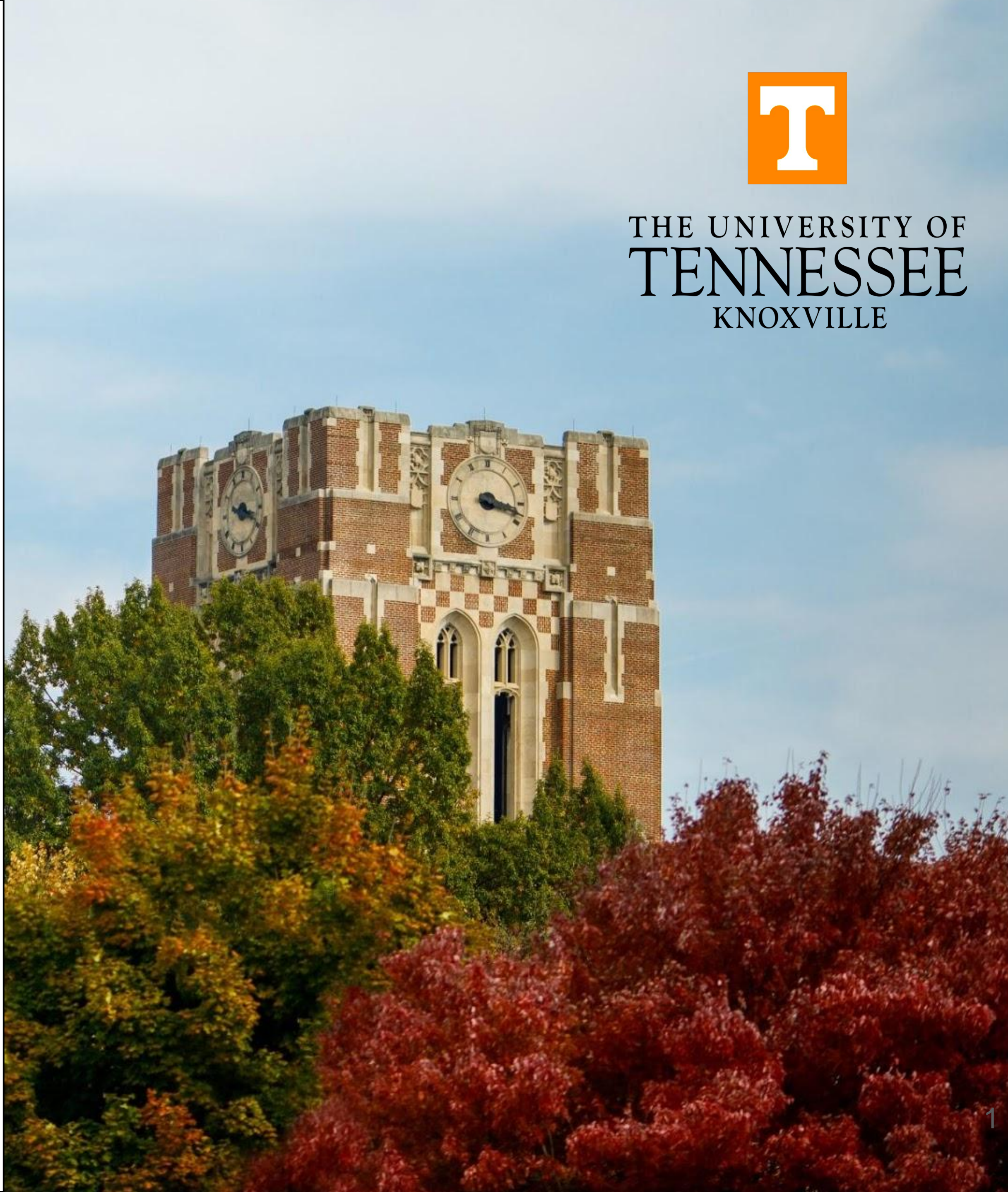


Outline:

- Motivation
- Introduction
- Quantum Algorithms:
  - QITE
  - QLanczos
  - Trotterization
- System
- Implementation of algorithms
- How to reduce errors?
- Results

Motivation

Neutrinos has been a big focus of research since it was discovered that **they are massive**. A lot of experiments support this claim:

- Solar Neutrinos, SNO Collaboration [PRD 17, 2369 (1978)]
- Evidence for Oscillation of Atmospheric Neutrinos, Super-Kamiokande Collaboration [PRL 81, 1562 (1998)]
- Discovery of Neutrino Oscillations [JASB 86(**4**), 303321. (2010)]

- Neutrinos oscillate collectively and exist in large numbers - a daunting task for a classical computer.
- The first attempt to study neutrinos using quantum computer was done Noh *et. al.* [NJP 14 033028 (2012)], and followed by many others [PRR 1, 033176 (2019); PRD 99, 123013 (2019); PRD 100, 083001 (2019)]

Introduction

Study: Collective Neutrino Oscillations (energy eigenvalues, transitional probabilities)

→

Using: QLanczos (which is based on QITE) Algorithm and first-order trotterization

→

On: IBM Q Hardware

- Hamiltonian can be separated into smaller blocks
- Represented using fewer qubits than those needed for the entire system
- Using isometry function to reduce circuit depth

Quantum Imaginary Time Evolution (QITE) and Quantum Lanczos (QLanczos) Algorithms

- Developed by Motta, *et al*. [Nat. Physics 16, 205-210 (2020)]

- **GOAL:** To determine ground and excited states of a system.

- **Why better than classical?**
  - Require exponentially less space and time per iteration,
  - Does not require ancillae, or high-dimensional optimization

QITE Algorithm:

Consider an arbitrary state: $|\Psi_0\rangle$

The evolution this state is given by:

$$|\Psi(\beta)\rangle = c_n(e^{-\beta H})^n |\Psi_0\rangle$$

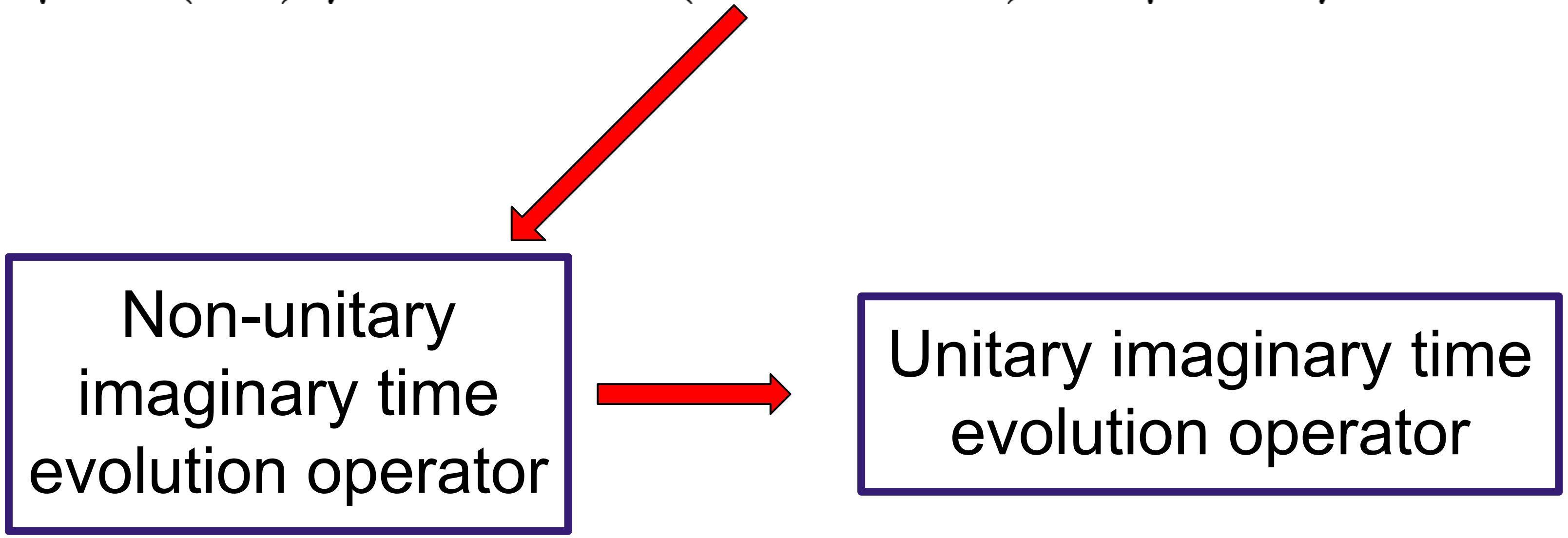
Normalization constant

Non-unitary imaginary time evolution operator

QITE Algorithm:

The evolution of an initial state is given by:

$$|\Psi(\beta)\rangle = c_n(e^{-\beta H})^n |\Psi_0\rangle$$



QITE Algorithm:

Divide the imaginary-time evolution into n small steps, $\Delta\tau = \frac{\beta}{n}$

Then, the sth step is given by: $|\Psi_s\rangle \propto e^{\Delta\tau H} |\Psi_{s-1}\rangle \approx e^{-i\Delta\tau A[s]} |\Psi_{s-1}\rangle$

$$A[s] = \sum_{i_0,\dots,i_{n_q-1}} a_I[s]\sigma_I$$

A general operator of Pauli strings

$$(\boldsymbol{S} + \boldsymbol{S}^T).\boldsymbol{a} = \boldsymbol{b}$$

Solving the linear system of equations

We try to find coefficient so that it matches with the non-unitary operator

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QLanczos Algorithm:

Each step of QITE → Gives energy expectation values → Input for QLanczos

- The Krylov space, is spanned by a set of vectors:

$$\{|\Phi_0\rangle, |\Phi_2\rangle, ...\}, |\Phi_l\rangle = c_l e^{-l\Delta\tau H} |\Psi[s]\rangle$$

$|\Psi[s]\rangle$: state at s-th QITE step

- Build Hamiltonian and overlap matrices:

$$\mathcal{H}_{l,l'} = \langle\Phi_l| H |\Phi_{l'}\rangle$$

$$\mathcal{T}_{l,l'} = \langle\Phi_l|\Phi_{l'}\rangle$$

QLanczos Algorithm:

- Express in terms of energy expectation values obtained from measurements on quantum computer:

$$\mathcal{T}_{l,l'} = \langle \Phi_l | \Phi_{l'} \rangle = \frac{c_l c_{l'}}{c_r^2}$$

$$\frac{1}{c_{r+1}^2} = \frac{\left\langle \Phi_r | e^{-2\Delta\tau H} | \Phi_r \right\rangle}{c_r^2}$$

Normalization constants

$$\mathcal{H}_{l,l'} = \langle \Phi_l | H | \Phi_{l'} \rangle = \mathcal{T}_{l,l'} \langle \Phi_r | H | \Phi_r \rangle$$

- Solve the generalized eigenvalue equation: $\mathcal{H}x = E\mathcal{T}x$

$$|\Psi[E]\rangle = c_E \left(x_0^{(E)} |\Phi_0\rangle + x_1^{(E)} |\Phi_2\rangle + \cdots \right)$$

an approximation to eigenvectors of Hamiltonian

First-order Trotterization:

- Decompose unitary time evolution such that:

Unitary time evolution operator → Operators - expressed in terms of one- and two-qubit operators on the quantum computers

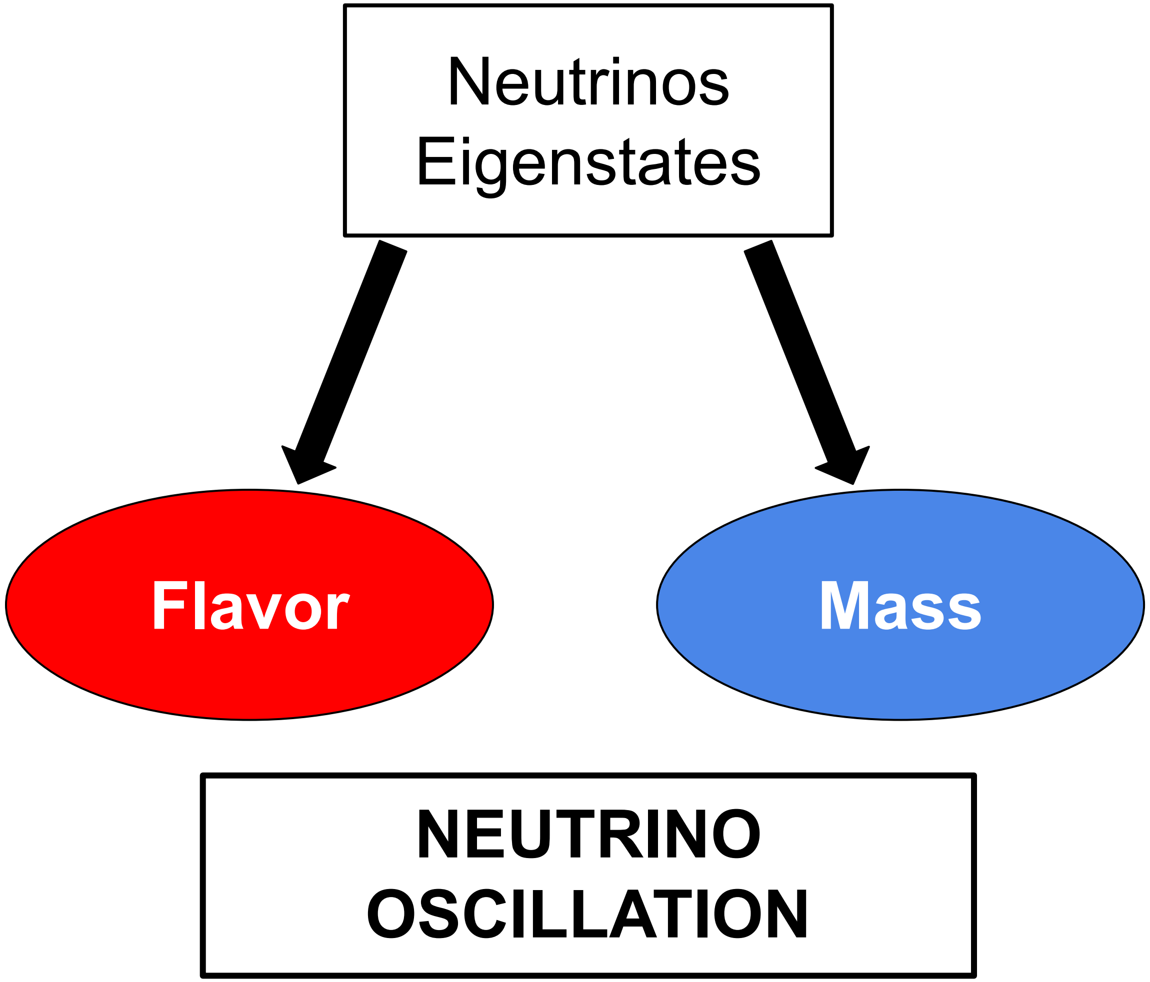
GOAL: To calculate the transition probability amplitude of collective neutrino flavor oscillations

Neutrino Oscillations

Neutrinos:

- Elementary particles
- Fermions
- Electrically neutral
- Exist in 3 leptonic flavors:
  - electron
  - muon
  - tau

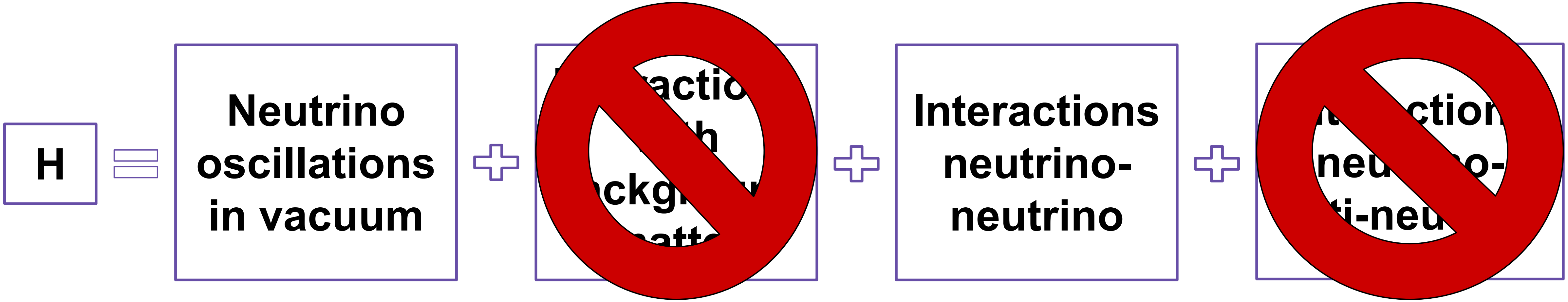
https://home.cern/science/physics/standard-model



[JETP, Vol. 26, p.984 (1968)]

Collective Neutrino Oscillations

Neutrinos experience self-maintained coherent oscillations - collective neutrino oscillations [Annu. Rev. Nucl. Part. Sci. 60:1, 569-594 (2010)]



Consider only 2 flavor/mass

$$H = -\frac{1}{2}\sum_{p=1}^{M}\omega_p Z_p + \frac{\mu(r)}{4}\left(\mathbf{X}^2 + \mathbf{Y}^2 + \mathbf{Z}^2\right)$$

[PRD 99, 123013 (2019)]

Collective Neutrino Oscillations

$$H = -\frac{1}{2}\sum_{p=1}^{M}\omega_p Z_p + \frac{\mu(r)}{4}(\mathbf{X}^2 + \mathbf{Y}^2 + \mathbf{Z}^2)$$

- H commutes with total number operator ($\mathcal{N}$)
- $\mathcal{N}$ has N+1 distinct eigenvalues: 0, 1, 2…., N
- H can be split into N+1 blocks which can be studied independently

We study: **4 Neutrino System** - **can split into 5 independent blocks**

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Four Neutrino System

$|0000\rangle$ → 0 particle

$|1111\rangle$ → 4 particles

2-Qubit systems corresponds to

$\{|1000\rangle, |0100\rangle, |0010\rangle, |0001\rangle\}$ → 1 particle

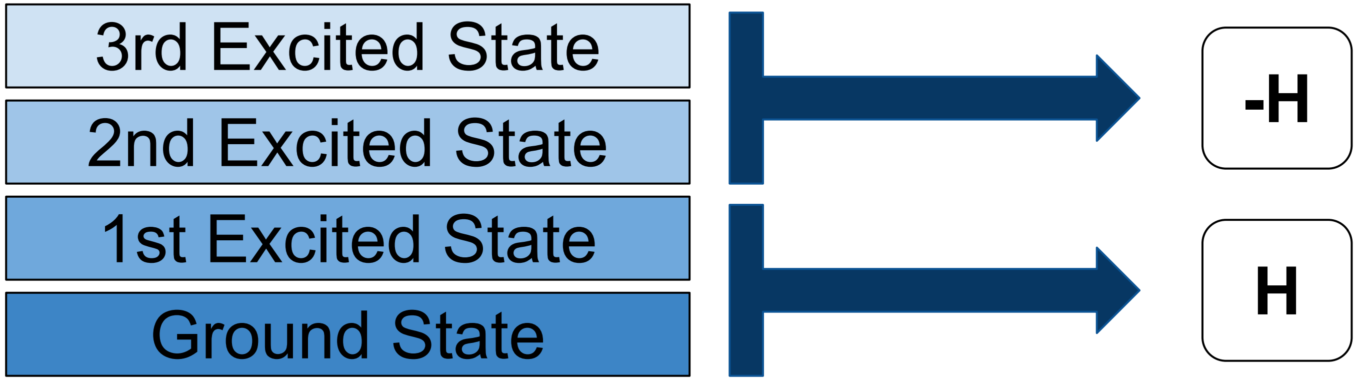
$\{|1110\rangle, |1101\rangle, |1011\rangle, |0111\rangle\}$ → 3 particles

3-Qubit systems corresponds to

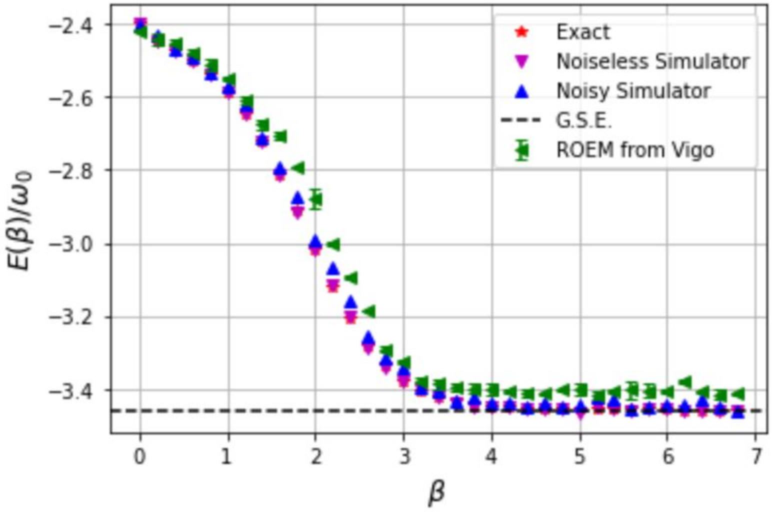
$\{|1100\rangle, |0011\rangle, |1010\rangle, |0101\rangle, |1001\rangle, |0110\rangle\}$ → 2 particles

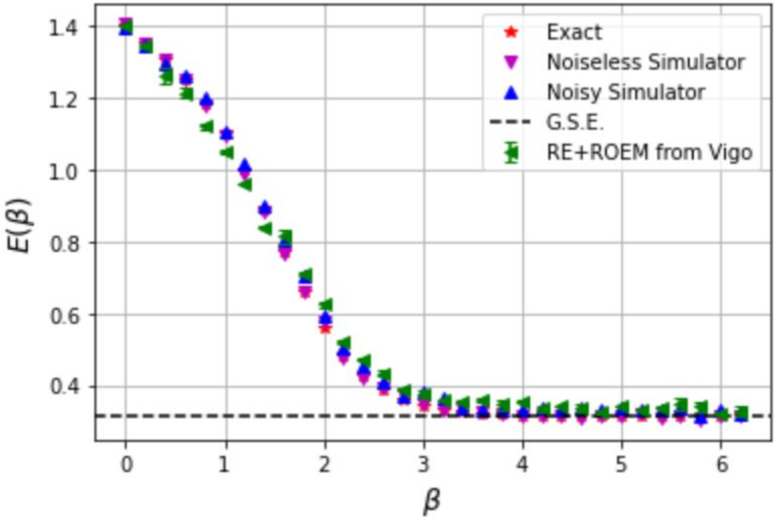
Four Neutrino System

- 2 Qubit system



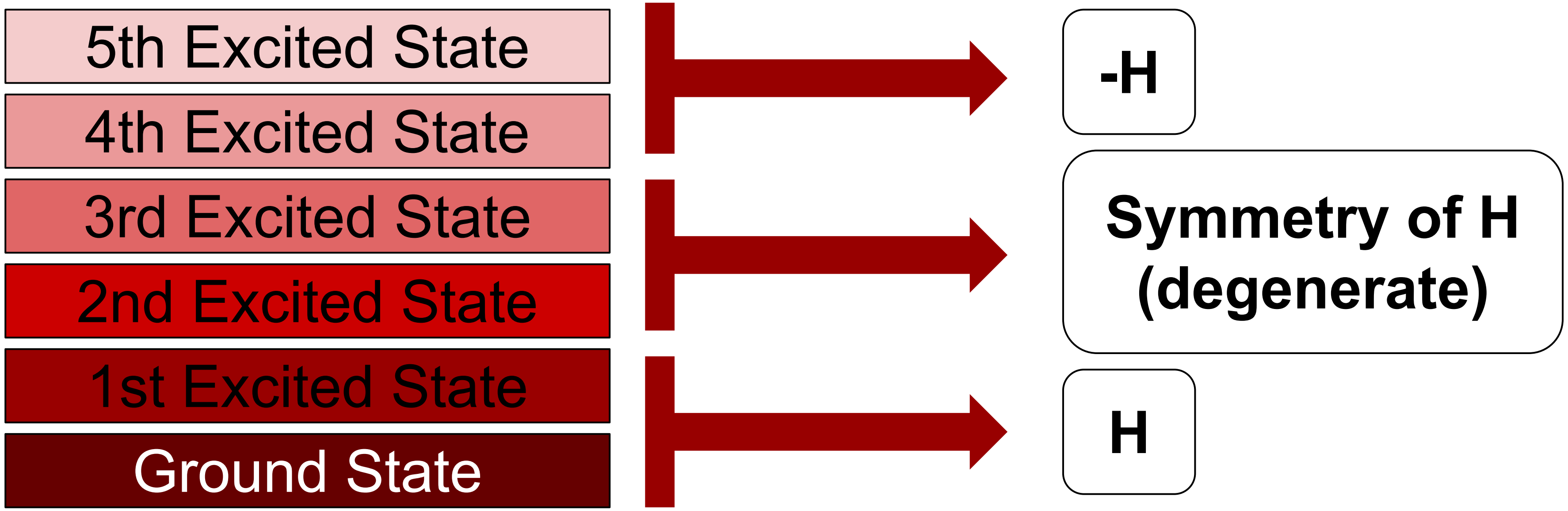
Example: Consider $\mu(r)/\omega_0$ = 0.2, exact eigenvalues are {-0.31, -1.39, -2.43, -3.45}





Four Neutrino System

- 3 Qubit system:



Implementation of QITE and QLanczos:

Determine the QITE steps that yields expectation value of energy within ∈=0.001 of ground state

→

Run the algorithm on IBM Q AER QASM Noiseless

→

Run the noisy simulator and get the the matrix element of S and b at each step

↓

Using the matrices from noisy simulator run the algorithm on IBM Q Hardware

N_{runs} = 2 runs
N_{shot} = 8192 shots per run

2-Qubit: Athens, Casablanca, Ourense, Rome, Santiago and Vigo
3-Qubit: Athens, Casablanca and Lima

For first excited state, use the QLanczos Algorithm

↓

At last choose the eigenvalue corresponding to total minimum uncertainty.

Implementation of first-order Trotterization:

Start with flavor Hamiltonian

with evolution given by:

$$\mathcal{U} = e^{-iH_{\text{flavor}}t}$$

$$H_{\text{flavor}} = -\frac{1}{2}\sum_{p=1}^{M}\omega_p(\cos 2\theta Z_p - \sin 2\theta X_p) + \frac{\mu(r)}{4}(\boldsymbol{X}^2 + \boldsymbol{Y}^2 + \boldsymbol{Z}^2)\ ,$$

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Implementation of first-order Trotterization:

Start with flavor Hamiltonian with evolution given by:

$$\mathcal{U} = e^{-iH_{\text{flavor}}t}$$

→

Split the Hamiltonian into three parts of commuting matrices

→

Use the first-order trotterization technique to get:

$$H_X = \frac{\sin 2\theta}{2} \sum_{p=1}^{M} \omega_p X_p + \frac{\mu(r)}{4} \boldsymbol{X}^2$$

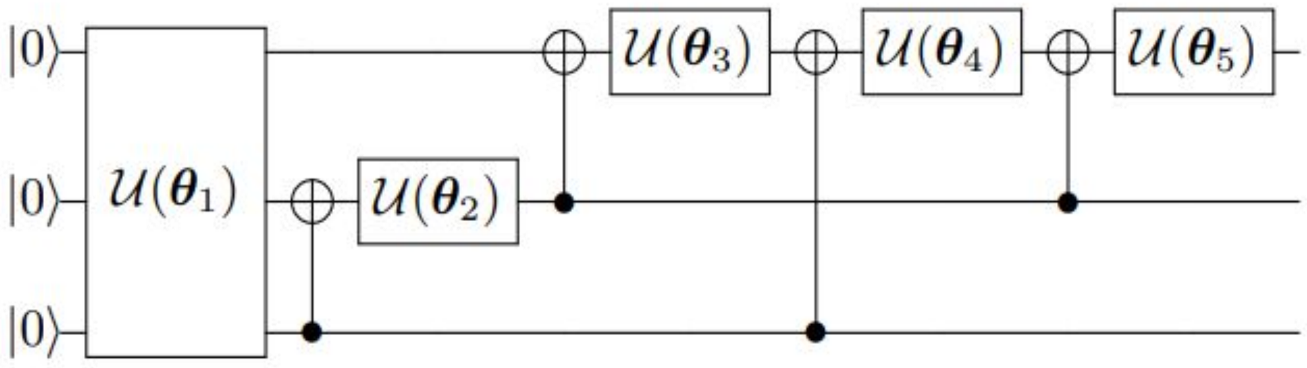
$$H_Y = \frac{\mu(r)}{4} \boldsymbol{Y}^2$$

$$H_Z = -\frac{\cos 2\theta}{2} \sum_{p=1}^{M} \omega_p Z_p + \frac{\mu(r)}{4} \boldsymbol{Z}^2$$

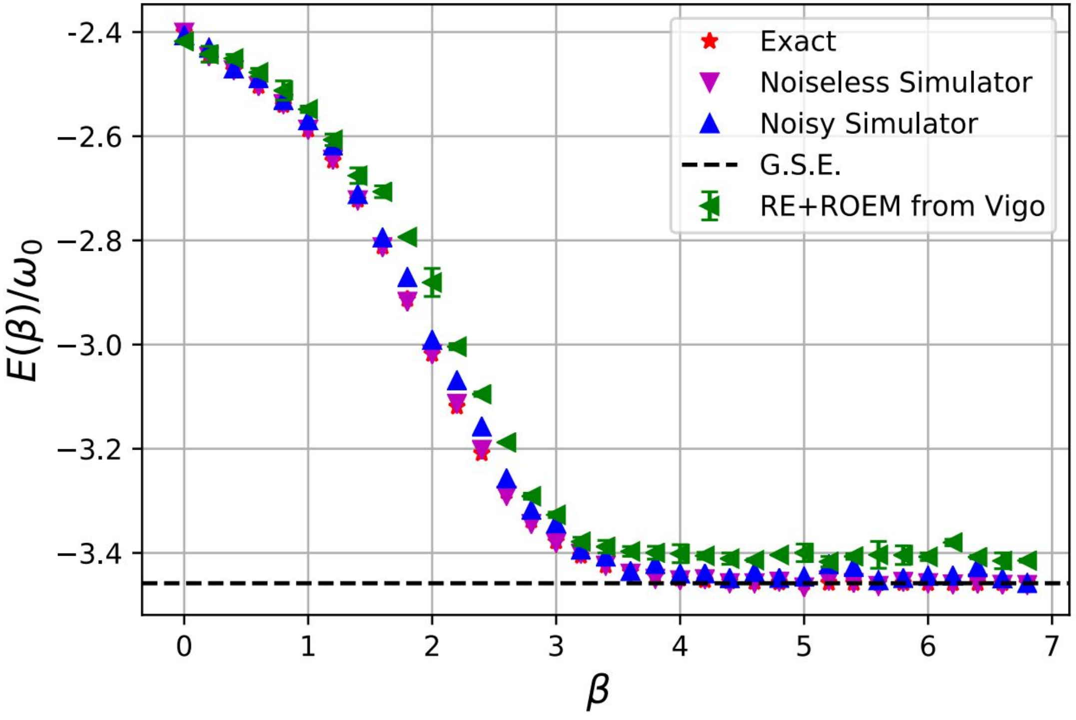
$$\mathcal{U} \approx \left[e^{-iH_X \Delta t} e^{-iH_Y \Delta t} e^{-iH_Z \Delta t} \right]^n$$

How to reduce error?

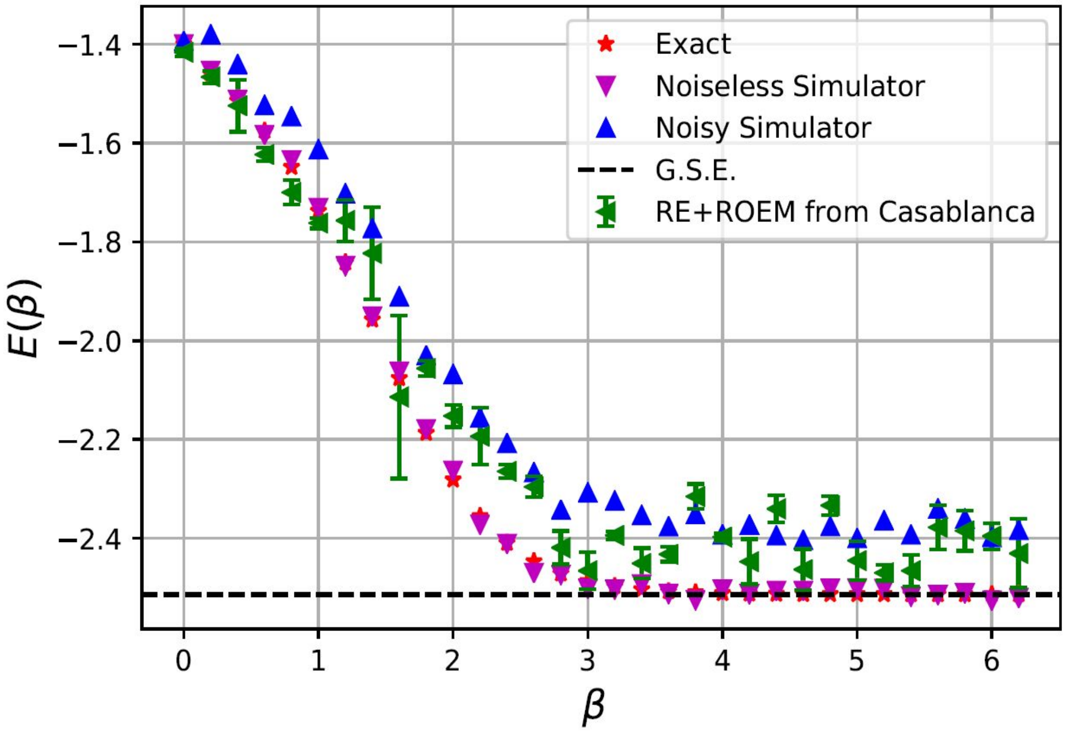
- Reduce the Hamiltonian in smaller blocks
- Use the properties of Hamiltonian to reduce number of measurements (consider only real elements, symmetry)
- Error Mitigation Techniques: Readout Error Mitigation (ROEM) and Zero Noise Extrapolation (ZNE)
- Used the *isometry* function (from Qiskit library) to shorten the depth of circuit



Results:



2-qubit plot: $\mu(r)/\omega_0$ = 0.2, and using H. The energies converge to the ground state energy level -3.45



3-qubit plot: $\mu(r)/\omega_0$ = 0.2, and using -H. The energies converge to the ground state energy level -2.51

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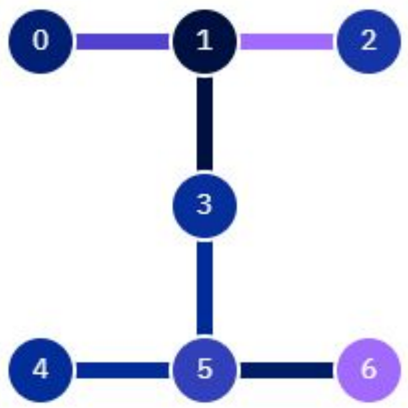
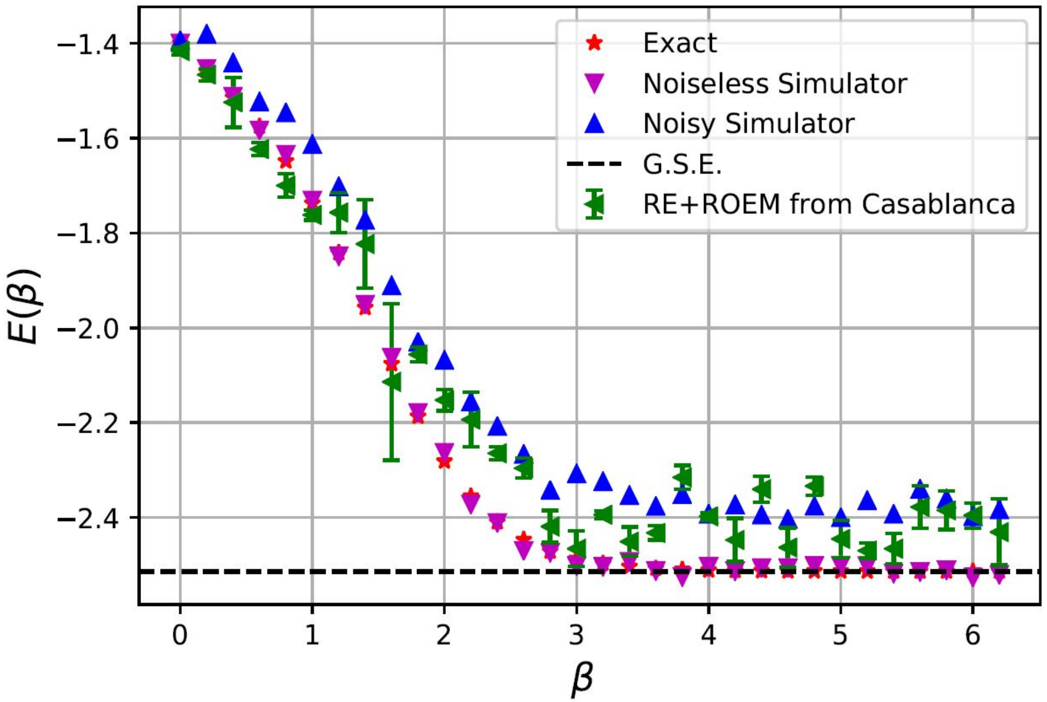
Results:



IBM Q Hardware - CASABLANCA

https://quantum-computing.ibm.com



3-qubit plot: $\mu(r)/\omega_0 = 0.2$, and using -H. The energies converge to the ground state energy level -2.51

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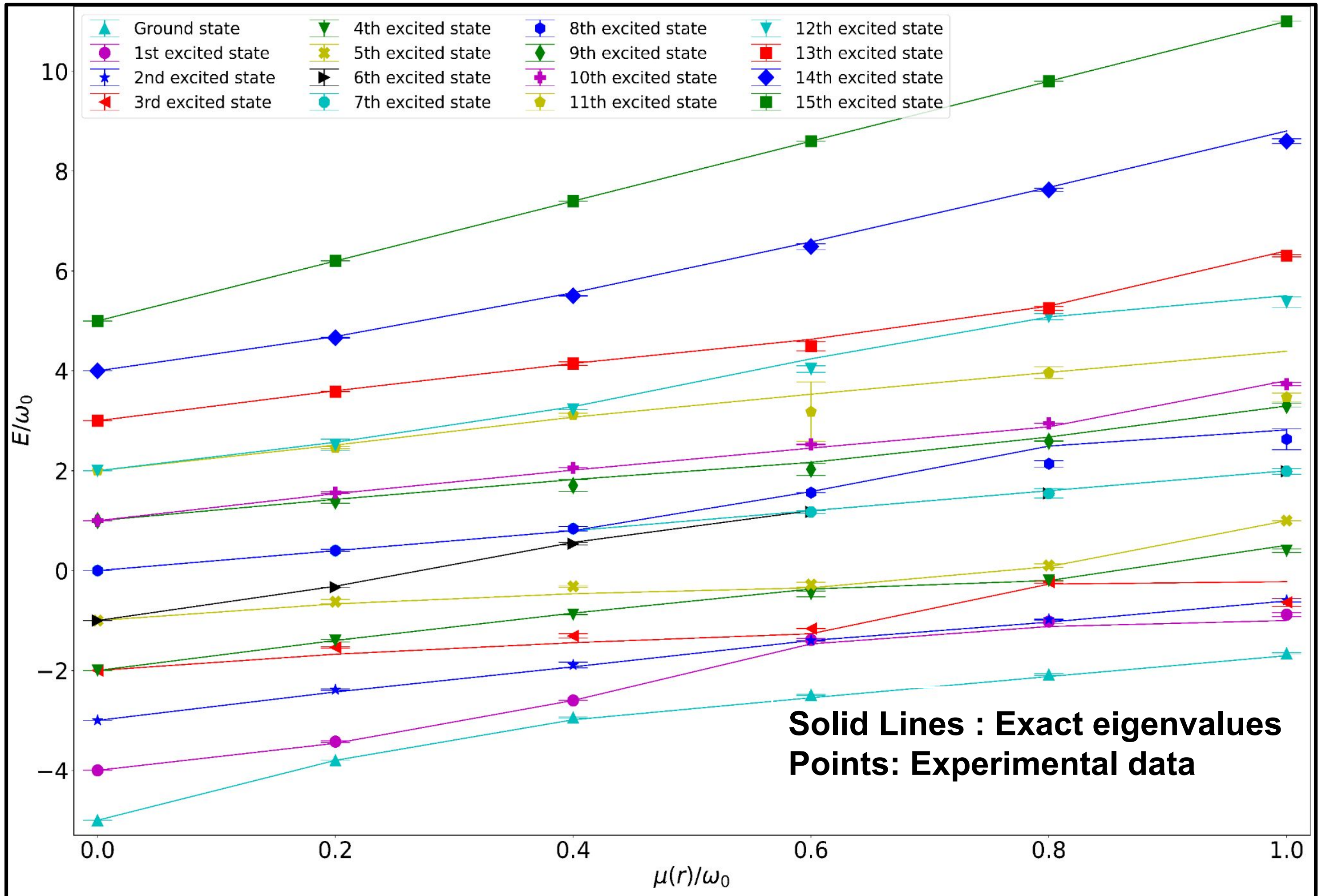
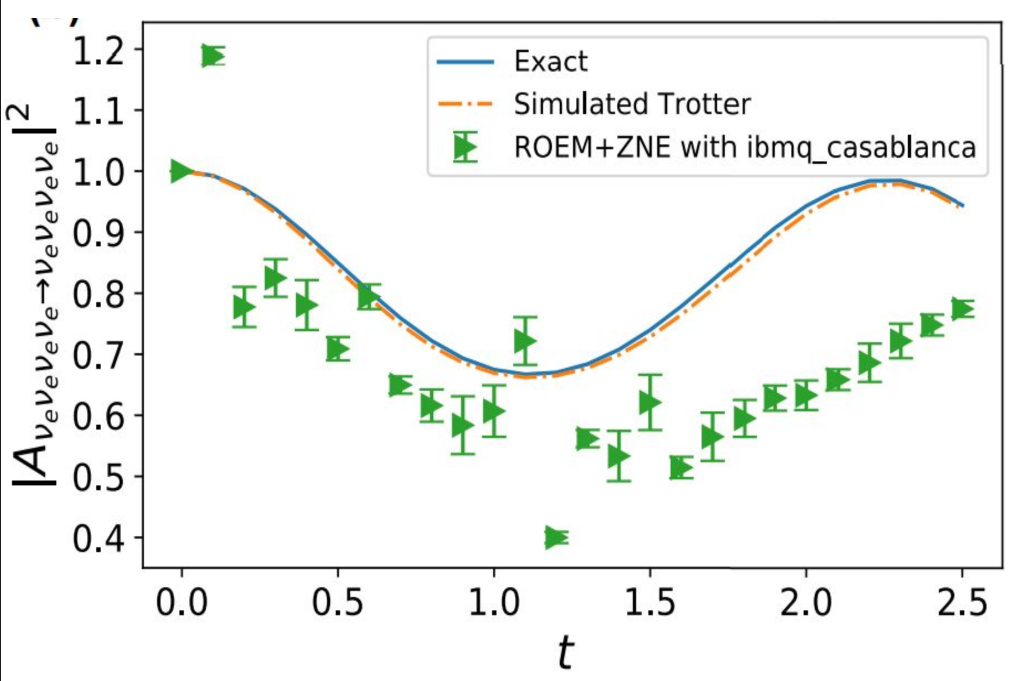
Results:



Energy eigenvalues of 4-neutrino system as a function of the radial dependence of the coupling system $\mu(r)/\omega_0$



Transition probability amplitude for $\mu(r)/\omega_0 = 1$, transition from

$$|\nu_e\nu_e\nu_e\nu_e\rangle \rightarrow |\nu_e\nu_e\nu_e\nu_e\rangle$$

Acknowledgements

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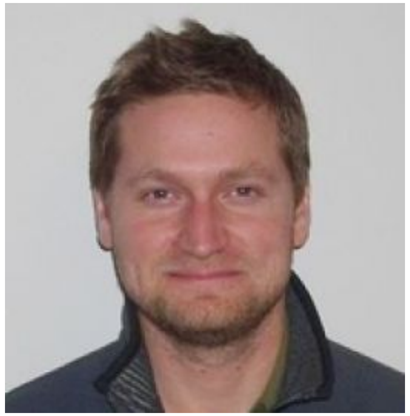

Dr. Raphael Pooser

Thank You